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ROSS'S

PRACTICAL SOLID GEOMETRY

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Drawing, Geometrical
Projection

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A MANUAL

OF

PRACTICAL SOLID GEOMETRY.

ADAPTED TO THE REQUIREMENTS OF

MILITARY STUDENTS AND DRAUGHTSMEN.

COMPILED BY

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PREFACE.

A FAMILIARITY with the principles and methods of Geometrical Drawing is essential for all those who would wish to represent, or record, on paper, the appearance, or proportions, of works designed by the architect and engineer. This little manual is compiled for the use of those who may wish not only to obtain a theoretical knowledge of that branch of Geometry which is usually termed Solid, or Descriptive, Geometry, but who may desire also to have the power of producing accurate and workmanlike drawings. The subject of Plane Geometry, also necessary to the draughtsman, has not been treated in this manual, as there already exist manuals of this branch of Geometrical Drawing which are within the reach of all. Up to the present, however, the treatises that have appeared on the subject of Solid Geometry have been either too elaborate, or too high-priced, to command any but a limited circulation. The object of the present work is to supply at a small cost the most essential and practical parts of what can be made a very complicated subject; in knowledge of which we in England compare unfavourably with our continental neighbours.

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Although of use to, and containing information for, all draughtsmen, this work is more especially intended to illustrate those parts of Solid Geometry which are of practical use to the military draughtsman. With this intention it follows in the main the lines of the course of Geometrical Drawing as studied at the Royal Military Academy, Woolwich. The convention, or modification of Orthographic Projection, known as the "System of Vertical Indices," being especially useful for the purposes of the military designer and draughtsman, has here received more attention than other methods; and the section relating to Defilade, although of no great extent, will be found in few other English works. Of this section of Solid Geometry the more practical and generally useful problems only are given.

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PRACTICAL SOLID GEOMETRY.

THIS term comprehends the investigation and application of those geometrical principles and methods by which points, lines, and bodies of three dimensions may be correctly represented by points, lines, and figures, "projected," or delineated, on a single plane surface. The plane surface, on which such delineations are made by the draughtsman, is the sheet of paper on which he works. The general method by which these representations of the originals are produced is technically termed "Projection." Projection is of three kinds: Radial or Perspective Projection, Orthographic or Perpendicular Projection, and Isometric Projection.

For a general representation of an object a perspective view, corresponding with the image made by the object on the retina of the eye of the observer, is, in many cases, suitable enough. In perspective projection the lines, or visual rays, proceeding from all points of the original object, are supposed to converge in a single point, the "vertex," which is usually taken to be the eye of the observer. These visual rays, being intercepted by the interposition of a plane surface (commonly taken vertical) and the points obtained on the plane by the impact of the visual rays, being connected with each other in a manner and in an order similar to the manner and the order existing in the original object, a representation is produced of that original object which is called a perspective projection or picture.

But this representation is much distorted when compared with the actual form of the original object; for not only are certain lines in the representation, corresponding with certain original lines of the object, "foreshortened" in different degrees, but the relative dimensions are still further affected by the varying distances of the original lines from the vertex towards which the visual rays converge.

The latter of these two elements of distortion can, however, be eliminated if the vertex be assumed to be infinitely distant, so that in the limiting position all parts of the object represented may be assumed to be equally distant from the vertex. The visual rays, or "projectors," on such an assumption, cease to be convergent, and become parallel one with another.

A projection or representation of an object under the condition of parallelism of the projectors, is termed an "Orthographic" projection; for the engineering draughtsman it is that form of projection which is of the greatest technical convenience, and of the most practical utility; and the following pages are, in the main, devoted to an elucidation of its principles, and to a description of its practical methods.

Note.—Isometric projection, it may be here noticed, is also of considerable practical use to the draughtsman; but it is only a special and modified form of orthographic projection. It will be sufficient, therefore, to give some short description of it after the details of orthographic projection have been explained.

Orthographic Projection.

All objects or bodies of three dimensions are technically called "solids," although, in the ordinary acceptation of that word, there may be nothing very solid about them. A geometrical solid may be considered to consist of a certain portion of space enclosed by surfaces, such surfaces being composed of an aggregation of lines, and such lines being, in each individual case, made up of points. It is assumed that the student, beginning a course of Practical Solid Geometry, is acquainted with the ordinary mathematical definitions of a point, a line, and a plane, and that it is not necessary to re-define these elements before proceeding to a closer examination of the methods and principles of Orthographic Projection.

Points, Lines, and Planes.

POINTS.

It is evident that the position of any point, floating (if the word be permitted) in space, is rigidly determined if its relation to any two fixed planes be known. In orthographic projection two planes are used, and, for convenience sake, are assumed to be the one vertical, and the other horizontal; these planes therefore are at right angles to each other. By imagining that the point in space (which may be called the "original" point) is connected with these planes by means of a line drawn perpendicular to each of them, we evidently rigidly fix or determine the position of the point.

Def. (1). The two planes are called "Co-ordinate planes of projection," and are distinguished, the one by being termed the "Horizontal plane of projection," and the other the "Vertical plane of projection."

Def. (2). The point in space is called the "Original point."

Def. (3). The perpendiculars drawn to the planes of projection and determining the point are called the "Projectors" of the point.

Def. (4). The points which are the intersections of the projectors with the planes of projection are termed "Projections" of the original point, and these are further distinguished by being called the "Plan," when the projection on the horizontal plane is referred to, and the "Elevation," when that on the vertical plane is intended.

Def. (5). The intersection of the co-ordinate planes is termed the "Axis," the "Ground line," the "Line of level," or, most commonly, the "*xy* line."

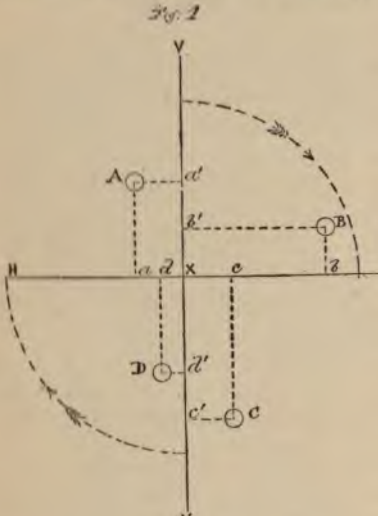
Note.—These terms are all used indifferently—the last mentioned is due to the custom that prevails of marking on drawings the line representing the intersection of the co-ordinate planes with the letters *x*, *y*.

Def. (6). The four right angles made by the intersection of the co-ordinate planes of projection are called "Dihedral angles."

From these considerations and definitions it follows—

That the distances between the plans of any series of original points are the true horizontal distances between those points in space, and that, similarly, comparisons of the elevations of any series of original points give the vertical relations, or differences of level, between those points. (Axiom 1.)

Fig. 1



The accompanying figure (Fig. 1) will serve to illustrate the matter at this stage.

Let **HH** and **VV** represent an end view of a horizontal and a vertical plane, and **X** the end view of the line in which they intersect.

Let **A**, **B**, **C**, **D** be four points in space, respectively occurring in each one of the dihedral angles formed by the intersection of the horizontal and vertical planes.

By drawing perpendiculars, **A***a*, **A***a′*; **B***b*, **B***b′*; &c., from each of the points **A**, **B**, **C**, **D** to both the horizontal and the vertical planes, we obtain the following results—

$$\left.\begin{matrix}a\\b\\c\\d\end{matrix}\right\}\text{ are plans and }\left\{\begin{matrix}a'\\b'\\c'\\d'\end{matrix}\right\}\text{ are elevations of }\left\{\begin{matrix}\mathbf{A}\\\mathbf{B}\\\mathbf{C}\\\mathbf{D}\end{matrix}\right.$$

And here, before going further, it is necessary to insist on the advantage of using a clear and consistent system of notation. In the notation which has been employed in this figure the capital letters **A**, **B**, &c., are used to denote original points (Def. 2), while *a*, *b*, &c., are used to denote their plans (Def. 4), and a', b', &c., their elevations (Def. 4). Inasmuch as the number of vertical planes, with regard to any one horizontal plane, is theoretically unlimited, the corresponding elevations on vertical planes other than the first, will have to be lettered a'' a''', &c., b'' b''', &c., &c.

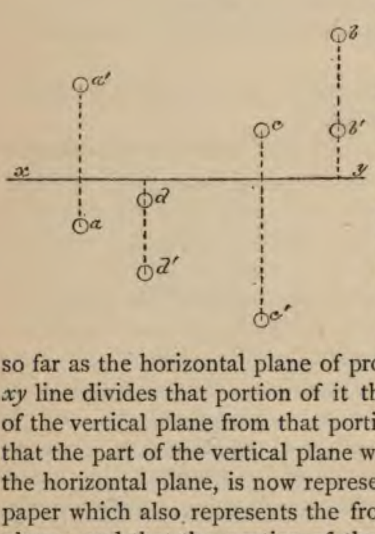
Note.—Any other logical system of notation may be used with equal advantage; that just stated is the one which has been employed by a great number of geometricians, and it therefore deserves consideration.

The point **A** may consistently also be termed the point (aa'), the point **B** (bb'), and so on.

The view of the horizontal and vertical planes of projection given in Fig. 1 is, however, only one particular appearance of those planes as seen by an observer whose eye is in the prolongation of the line resulting from their intersection. Although such a view, in the case of the points shown in Fig. 1, gives us a considerable amount of information concerning those points, it still leaves uncertain the exact relative position of the original points.

Again, it is evidently impracticable to bend the drawing paper into the form of the co-ordinate planes, and to attempt the solution of problems, requiring great accuracy of manipulation, on such an arrangement of the paper. For the convenience of the draughtsman, therefore, some other conventional mode of representing the co-ordinate planes, which shall supply all the necessary information, and enable him to work on the paper with the greatest amount of ease and accuracy, must be adopted. Referring to Fig. 1, if the observer of the system of points shown in that figure be supposed to be placed above the horizontal plane, and if the vertical plane itself be made to revolve on the axis of the co-ordinate planes, till it become coincident with the horizontal plane of projection, it will be seen that, at any rate, the plans and elevations of all the original points will be before him delineated upon a plane surface. And this is the convention used in orthographic projection, the direction in which the vertical plane of projection is revolved being shown by the arrows given in Fig. 1. In Fig. 2 is given a representation of the system of points, **A, B, C, D,** shown in Fig. 1 after the revolution of the vertical plane has taken place, and, as many important axioms result from a consideration of this figure, the student will do well to thoroughly master the lessons it conveys. From Fig. 2 it will be seen that the line *xy* represents the axis of the co-ordinate planes; that, so far as the horizontal plane of projection is concerned, the *xy* line divides that portion of it that was originally in front of the vertical plane from that portion of it that was behind; that the part of the vertical plane which originally was below the horizontal plane, is now represented by that part of the paper which also represents the front part of the horizontal plane; and that the portion of the vertical plane which was originally above the horizontal plane coincides in surface with the back part of the horizontal plane. Hence—

Fig 2



Any point *above* the horizontal plane, and *in front* of the vertical plane (as **A**), will have its plan *below* the *xy* line, and its elevation *above* that line; that is to say, the plan and the elevation will be on *opposite* sides of the *xy* line.

Note.—"Above" and "below," as regards the *xy* line, are merely relative terms referring to position on the paper.

Any point (as **B**) *above* the horizontal plane, and *behind* the vertical plane, will have the plan and elevation both *above* or on the *further* side of the *xy* line.

Any point (as **C**) *below* the horizontal plane, and *behind* the vertical plane, will have its plan *above* and its elevation *below* the *xy* line.

Any point (as **D**) *below* the horizontal plane, and *in front* of the vertical plane, will have its plan and elevation both *below* the *xy* line.

The following axioms are derived from a consideration of these facts :—

The perpendicular distances of any point from the horizontal and vertical planes of projection are, respectively, given by the perpendicular distances of its elevation and plan from the *xy* line. Axiom (2).

The plan and elevation of every original point lie, on the surface of the paper, in a line at right angles to the ground, or *xy*, line. Axiom (3).

An original point actually lying on the horizontal plane has its elevation in the *xy* line. Axiom (4).

Conversely, an original point actually situated in the vertical plane has its plan on the ground line. Axiom (5).

An original point actually situated in the ground line has neither plan nor elevation separate from itself; that is to say, the original point, its plan, and its elevation, are all coincident. Axiom (6).

It will readily be seen that, given the plans of any series of points, and their elevations with regard to any given *xy* line representing the position of the vertical co-ordinate plane, it is a simple matter to obtain their elevations with regard to any other vertical plane. For, given the ground line of the new vertical plane, it is only necessary to draw projectors from the plans of the points perpendicular to the new ground line, and to measure off on these, from the new ground line, the heights of the vertical distances which are given on the first vertical co-ordinate plane. Care must be taken to "set off" or "plot" these distances respectively on the near, or farther, side of the new *xy* line, according as the original points happen to be below, or above, the horizontal plane to which the plans are referred.

LINES.

All lines, being composed of a collection, or aggregation, of points, their projections will consist of the aggregations of the projections of all the points of which they are composed. Axiom (7).

Note and Def. (7).—The aggregation of the projectors of a collection of points composing a line is termed the "projecting surface" of that line, and when the original line is a straight line, that surface is a plane surface.

It follows, therefore—

That the projections of all original straight lines must be themselves straight lines. Axiom (8).

That a straight line, parallel to both of the co-ordinate planes, has both of its projections parallel to the *xy* line. Axiom (9).

That a straight line, parallel to one of the co-ordinate planes, has its projection on the other parallel to the *xy* line. Axiom (10).

That a straight line perpendicular to either of the co-ordinate planes has, for its projection on the plane to which it is perpendicular, a point only; and that its projection on the other co-ordinate plane is a line at right angles to the *xy* line. Axiom (11).

That the projections of a line passing through the *xy* line will meet on the ground line in that point in which the original line traverses the ground line. Axiom (12).

That all original lines, not parallel to either of the co-ordinate planes, must, if sufficiently produced, meet both of the co-ordinate planes. Axiom (13).

Def. (8). The points in which an original line traverses the co-ordinate planes are termed its "Traces"; and these are further distinguished by being called "horizontal" and "vertical" traces, according as the trace, in each case, occurs on the horizontal, or the vertical, co-ordinate plane.

With regard to these traces it is evident, therefore, that—

For a line parallel to both of the co-ordinate planes there is no trace. Axiom (14).

For a line parallel to one of the co-ordinate planes there is only one trace, and this one trace is to be found on the other co-ordinate plane. Axiom (15).

For a line not parallel to either of the co-ordinate planes there is a trace on each. Axiom (16).

For a line traversing the ground line of the co-ordinate planes the traces are coincident with the point in which the ground line is traversed by the line. Axiom (17).

Note.—Compare Axiom (12). The traces, the original point in which the line traverses the ground line, the plan, and the elevation of this original point are all represented by one point.

These propositions being granted, it follows that—

The length of the projection, on either of the co-ordinate planes, of any finite straight line depends on the angle which the line makes with that plane. Axiom (18).

Note.—To state the relation in mathematical terms ; if—

L be the length of the original line,
l ,, ,, ,, ,, its projection,
α ,, ,, angle made by the line with the plane,

then—

$$l = L \cos \alpha.$$

Further, if the line is parallel to the plane,

$$l = L.$$

PLANES.

Similar axioms may be deduced in the case of planes which are aggregations of straight lines connected one with another by some definite law and circumstances of generation.

Note.—It has been already assumed (p. 6) that the ordinary conception of a plane is familiar to the student, but, as all geometrical surfaces may be conceived as having been generated by the motion of a geometrical line, traversing or passing through space in conformity with some law or condition, it will be as well to give a more suitable definition of a plane.

Def. (9). A plane may be conceived as being generated by the motion of an indefinite straight line always meeting another, and moving parallel to a third straight line, both of these being fixed in position.

Note.—By placing two straight wires in any fixed positions, and by moving a third along one of these, in accordance with the above given conditions, the student's eye will readily conceive the existence of a plane so generated. For the construction of working models, or for the exemplification of solutions of problems, sheets of glass are excellent representatives of planes, due corrections being made for unnecessary thickness, in which attribute a geometrical or mathematical plane is entirely wanting.

Def. (10). The "traces" of a plane are the straight lines given by the intersection of the plane with each of the co-ordinate planes, and these traces are further distinguished (as in the case of lines) by being termed respectively "horizontal trace" and "vertical trace."

Note.—Since no two different planes can have both of these traces the same in each case, these traces are, when considered together, sufficient to distinguish any one plane from all other planes.

The following axioms are derived from a consideration of the relation of planes to the co-ordinate planes of projection:—

A horizontal plane can have only a single trace, and this trace will be a straight line on the vertical plane of projection, and parallel to the xy line. Axiom (19).

A vertical plane, when itself parallel to the vertical plane of projection, has only a horizontal trace, which is a straight line parallel to the xy line. Axiom (20).

All vertical planes (with the exception of that mentioned in the preceding axiom) have two traces, of which the trace on the vertical plane of projection is at right angles to the xy line. Axiom (21).

A plane perpendicular to the vertical plane of projection, provided that it is not a horizontal plane, for which see Axiom (19), has two traces, the horizontal trace being at right angles to the xy line. Axiom (22).

The traces of a plane perpendicular to both of the co-ordinate planes will both be at right angles to the xy line, and will be represented on the paper (on the assumption of the revolution of the vertical plane of projection), by a continuous line at right angles to the xy line. Axiom (23).

All other planes will have two traces which will meet, unless they are both parallel to the ground line, at a point on the ground line. Axiom (24).

The horizontal and vertical traces of all parallel planes are themselves respectively parallel. Axiom (25).

Note.—It seems unnecessary to give figures representing the theorems given above. If the student has any difficulty in following these propositions the use of two co-ordinate planes, made of wood, and pieces of glass or cardboard cut into various shapes, which can be placed in different positions with regard to the co-ordinate planes, will assist the labouring imagination.

The actual constructive operations based on these considerations and required of the draughtsman, although simple enough, are sufficiently confusing to the beginner. This circumstance is due to the difficulty experienced by most in exercising what may be called the pictorial imagination. To carry in the mind's eye a continuous picture of the constructions used and the results obtained during the working out of a problem is a faculty possessed by none in perfection, although in the case of some individuals it is more largely developed than in the case of others. By practice and cultivation it may be induced where wanting, and in all cases it is capable of being developed and improved.

Note.—No two persons can be treated alike by the instructor in his efforts to develop this most essential power, which has been termed the pictorial geometrical imagination. The use of models is absolutely necessary to enable some to comprehend illustrations and problems; but there are others who will more quickly, and to a greater degree, exhibit this faculty, if they are permitted to use models only rarely.

To reduce the labour and strain on the imagination, an important modification of the system, already explained, of orthographic projection has been put forward. This modified method of projection certainly simplifies the constructions required of a draughtsman, and, were this the only advantage to be gained by its use, it would be worthy of adoption. But it has other practical advantages, especially in the representation of ground and (as in the case of fortifications) of works adapted to actual sites whose surfaces consist for the most part of a combination of planes. This method has received more than one name, but it will here be consistently mentioned as the "Method of Vertical Indices," or (more briefly) the "Index System."

The Method of Vertical Indices,

OR, THE INDEX SYSTEM.

The principle of orthographic projection by the method of vertical indices is to deal directly with representations on only one of the co-ordinate planes, supplying the information required with regard to the other plane of projection, for each point, line, and plane, by means of a series of numbers which are called "indices." Either of the two planes might be used, and the surface of the paper on which the drawing is made being assumed as this plane, the indices would be entered as conveying facts regarding the other plane of projection. But owing to the fact that the surface of ground in nature is more generally horizontal in character than vertical, and that the works of man are, as in the case of habitations and works of fortification, more extensive and more complex in horizontal development than in vertical, it is customary to use the horizontal plane for actual representation, and to enter on the plan, according to some consistent system of notation, the information supplied by a vertical plane of projection. From the information thus supplied elevations on any vertical plane can be given, the position of the xy line being fixed as required. Till such elevations become necessary the "figured," or numbered, horizontal projections are in themselves capable of affording most of the information which can be deduced from a considerable number of vertical planes of projection.

Def. (11). A plan representation, figured with numbers giving the height of each point, line, surface, above the horizontal plane, is called a "Figured plan."

Def. (12). The number affixed to each point, line, or surface, is called the "Index" of that point, line, or surface, and usually relates to some fixed scale used in the delineation of the plan.

Note.—It is usual to distinguish points above and below the horizontal plane on which the plan is projected with the signs (+) and (−) respectively. The indices are usually written below the plan lettering when letters are used. It is also usual to omit (+) and only write (−) when required.

Def. (13). The horizontal plane is commonly called the "Original" or "Datum" plane, and it may be assumed at any height that may be considered convenient.

Note.—It is usual to assume the index of the datum as (± 0) in drawings which are referred to a theoretical plane in space. For drawings of ground or of works connected with the surface of the ground, the datum is often assumed to be the sea level.

On this system of representing the results of orthographic projection it follows that—

A point is fully determined if its plan is figured. Axiom (26).

A straight line is fully determined if the plans of two points in it are figured. Axiom (27).

All points in a horizontal line carry the same index. Axiom (28).

Note.—The inclination of a line to the horizontal plane is evidently obtained by taking the difference of the indices of any two points on it, and comparing that difference in level with the horizontal distance (obtained from the plan, and computed to the same scale of units as that used for the indices) between the two points. That is to say, if **N** units be the difference of the indices of any two points in the plan of the line, **M** the distance in units of the same scale between these points, (α) the angle of inclination of the line to the horizon, then

$$\tan(\alpha) = \frac{\mathbf{N}}{\mathbf{M}}$$

Def. (14). For the future the term "inclination" will be used to define the inclination of a line or a plane to the horizontal plane only.

In the case of planes a convenient convention is to deal with all those lines lying in the plane which are horizontal, each of which will evidently carry a single index. (See Axiom 28.)

Def. (15). These horizontal lines in any one plane are called the "Horizontals" of that plane. When these horizontals occur, successively, at equidistant vertical intervals they are also called "contours."

Note.—The word contour is of French origin, and the term is derived from the fact that it is usual to represent the configuration of undulating ground by the lines given by the intersections with the surface of the ground of a series of horizontal planes. The ground so represented is said to be "contoured."

It is evident that each horizontal is parallel to the horizontal trace of the plane. Axiom (29).

In the place of drawing a considerable number of such horizontal contours to represent the horizontal figured plan of a plane, it is sufficient to draw only a few (two being all that are absolutely required), and, in some convenient part of the drawing, to place across, and at right angles to these, a line, figuring this line, at the points it is intersected by the horizontals, with the indices of the contours.

Def. (16). A line drawn at right angles to the direction in plan of the horizontals and figured with their indices is called the "scale" of the plane.

Note.—Or, more fully, the "Scale of Slope" of the plane. To distinguish this scale from any other line lying in the plane it is customary to draw another line close to, and parallel to, the line of the scale; it is also of advantage to assist the observer in the reading of figured plans to thicken one of these lines. In accordance with the usual custom, that one of these two scale lines is thickened which is on the left hand of the observer as he looks, or reads, up the plane.

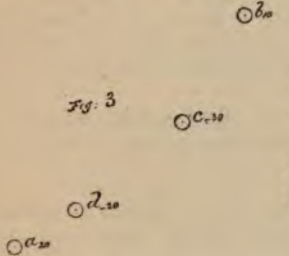
The "scale of slope" of any plane (and hence the usage of the term) enables us to state the inclination of the plane. By constructing a right-angled triangle whose height is equal to the difference in level between any two horizontals shown on the scale of slope, and whose base is equal (stated in units of the same scale as is used for the vertical indices) to the distance apart of those horizontals on the scale of slope, the inclination of the plane, being the angle contained by the hypothenuse and base of that right-angled triangle, is obtained.

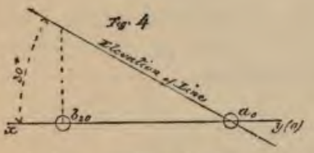
Note.—The same result is obtained if an elevation of the plane be made on a vertical plane whose horizontal trace is parallel to the scale of slope, or, in other words, at right angles to the horizontal trace of the plane. In the solution of problems dealing with planes, and where it is necessary, or of advantage, to make elevations of the data, it is a general rule that the elevations should be made on vertical planes parallel to the scales of the planes.

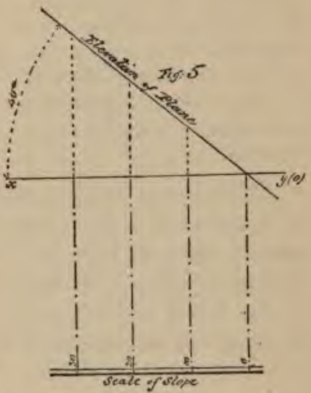
Any line lying in a plane which is parallel to the scale of slope of the plane has evidently the same inclination as the plane itself. Axiom (30).

If a line and a plane are perpendicular each to the other, the plan of the line is at right angles to the horizontals of the plane. (Axiom (31).

Figures 3, 4, and 5 represent points, lines, and planes,







treated according to the notation employed in the method of projection by indices.

Fig. 3 represents the same series of points that were displayed in Fig. 1 and Fig. 2.

Fig. 4 represents a figured line whose inclination is 30°.

Fig. 5 represents a plane whose inclination is 40°.

Note.—The scale to which all the indices in these figures are referred is one of 40 units to the inch.

Circles.

The orthographic projection of a circle may be either—

A circle of equal size; which is the case when the plane of projection is parallel to the plane of the original circle. Axiom (32).

A line equal in length to the diameter; which is the case when the plane of projection is at right angles to the plane of the original circle. Axiom (33).

An ellipse; when the plane of projection is oblique to the plane of the original circle. In such an ellipse the major axis is equal to the diameter of the circle, and the length of the minor axis depends on the obliquity of the plane of projection to the plane of the original circle. Axiom (34).

Note.—The major axis of the ellipse is the projection of that diameter of the original circle which is parallel to the plane of projection, and the minor axis is the projection of the diameter which is in the plane of the original circle at right angles to the diameter giving the major axis as its projection.

Note.—When two diameters of a circle mutually intersecting at right angles are projected, they become in the projected ellipse conjugate diameters.

Other curves so rarely occur in the solution of problems, or, at any rate, in such problems as are dealt with by the military student, that it is unnecessary to give exemplifications of them. If required, certain points on the curves can be taken, and the projection of the curve determined by fixing the projections of these points.

The Projection of Solids.

Before proceeding to the statement and resolution of the more commonly useful problems of solid geometry, it will be of advantage to give a more particular account of that sub-division of orthographic projection which is usually termed "The Projection of Solids."

The word "solid," as used in the term Solid Geometry, is employed (see p. 6) in a somewhat special and technical sense. But this term, although usually confined to a description of space included within surfaces, is also used to describe those forms which matter assumes, in some instances naturally, in others by artificial construction. Such bodies are solid in the more ordinary acceptation of the word, and being solid, and in varying degrees opaque, when modelled into geometrical forms, are of great assistance to the eye in working out problems of projection.

Such solids may be "regular" or "irregular" in their forms. The "regular" solids are those formed by the combination of equal and regular polygons, and each of them is capable of being circumscribed by a sphere. There are five such solids, the Tetrahedron, the Hexahedron or Cube, the Octahedron, the Dodecahedron, and the Icosahedron, and they are, with the exception of the cube, which is well known to everyone, all represented in plan

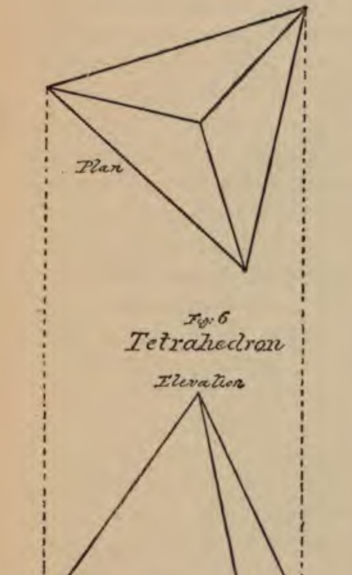


Fig. 6
Tetrahedron

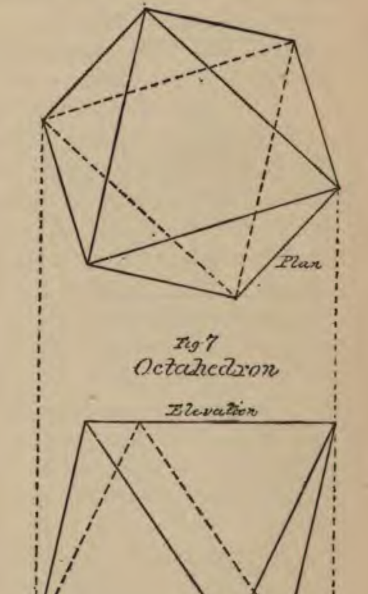
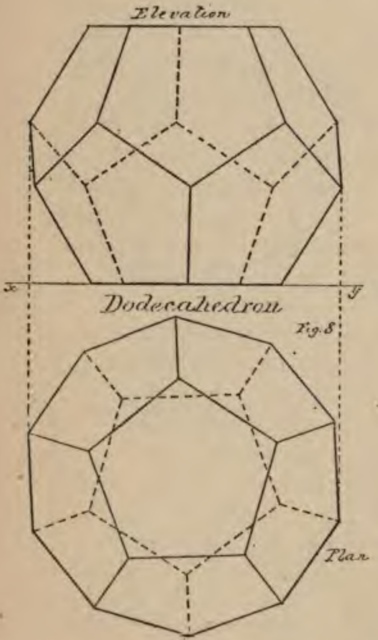


Fig. 7
Octahedron

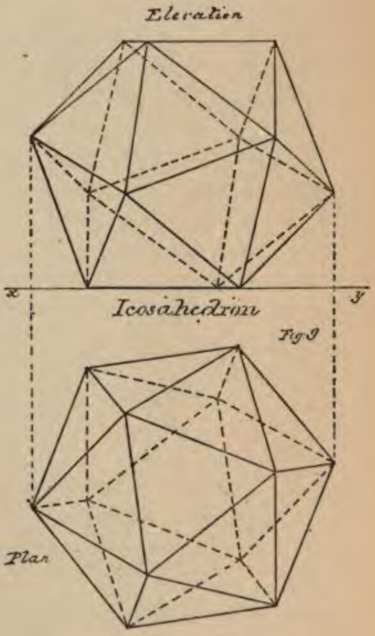
and elevation, when resting on one face on the horizontal plane, in Figs. 6, 7, 8, and 9.

The tetrahedron is comprised of four equal equilateral triangles, and its plan, when one face rests on the horizontal plane, is found by drawing an equilateral triangle of side equal to the stated length of the edge of the solid, and by drawing, to represent the slant edges of the solid, lines from the centre of the circle inscribed in that triangle, to each of the angles of the base.



Dodecahedron

Fig. 8



Icosahedron

Fig. 9

Note.—To figure, or index, the plan of the vertex, a right angled triangle, whose base is equal to one of the lines drawn from the centre to the angles of the equilateral triangle, and whose hypothenuse is equal to the real length of this line, will have to be drawn. The perpendicular of this triangle is the height of the vertex of the solid above the horizontal plane. This operation is the converse of that used when it is desired to ascertain the inclination of a given line. (*See ante*, p. 17.)

The cube consists of six equal squares, and its plan, when resting on one of its faces, is a simple square of the given dimension of the edge of the solid. The four angles of this square should be figured each with two indices, differing, in reading, by as many units of the scale employed as may be found in the length of the edge.

The Octahedron is comprised within eight equal equilateral triangles. When resting on one of these in the horizontal plane its plan is obtained by drawing this triangle and another concentric and alternating with it. By this construction the angles of one of these triangles give the three points of the solid resting on the horizontal plane, and the opposite upper horizontal face of the solid is given by the other triangle. To complete the plan of the solid it is necessary to connect alternate upper and lower points by lines representing the slant edges of the solid.

The Dodecahedron is contained by twelve equal and regular pentagons. The plan of the solid is obtained by drawing in the first instance two of these pentagons concentric and alternating with each other. One of these represents the lowest face resting on the horizontal plane, and the other the opposite face, parallel to the lowest, and at the extreme height of the solid. The remaining points of the solid are determined by drawing two larger pentagons each respectively concentric with and parallel to the lowest and uppermost pentagon. The side of each of these larger pentagons is equal to the diagonal of the smaller pentagon. By joining the ten points so obtained one by one successively round the figure and alternately to one of the angles of the two small pentagons the plan is completed.

The Icosahedron is contained by twenty equal equilateral triangles. To draw the plan of the solid draw two of the triangles concentric and alternating with each other; these will be the lowest and uppermost horizontal surfaces when the solid rests on the horizontal plane. On the given side of one of these triangles, or on a line equal to it, construct a pentagon, and, using the diagonal of this pentagon, draw alternating equilateral triangles of sides equal to this diagonal parallel to and concentric with each of the two first-mentioned equilateral triangles. The points of the solid being now all obtained, the plan is completed by drawing all the plans of the triangles of which the surface of the solid is composed.

For elevations of these solids resting as described on the horizontal plane it will be necessary to ascertain the heights of the various levels at which the points of the solids occur above the horizontal plane. The general principle on which this investigation is conducted has been indicated in the note attached to the description of the tetrahedron (*see* p. 23), but may now be briefly stated. To find the difference in level between any two points of an original line it is necessary to construct a right-angled triangle whose base shall be the distance apart in plan of the plans of the two points, and whose hypothenuse shall be the real length of the line. The resulting height of the perpendicular of this triangle will evidently be the difference of level between the two points. By drawing, when so ascertained, the vertical traces of the horizontal planes in which the points of the solid are situated with proper regard to the xy line on which an elevation is required, and by projecting the points concerned from the plan up to these planes of level, the elevations of those points are at once given, and the projection on the vertical plane of projection can be completed.

Note.—For the Tetrahedron and Octahedron one such constructive operation is sufficient to ascertain the height between the lowest and uppermost horizontal surfaces. For the Dodecahedron and Icosahedron two operations will be required. If, however, the plan and real length of an imaginary line extending from any point in the upper horizontal surface to any point in the lower horizontal surface be employed, the height between these surfaces can be obtained by a single construction.

Other solids which afford good exercises in projection are prisms, pyramids, spheres, cones, and cylinders.

Prisms and pyramids are solids contained by plane figures, and may be either regular or irregular as regards disposition of the surfaces of their faces, and either right or oblique as regards the relation of their axes to their bases.

Spheres, cones, and cylinders are solids of revolution—a term used to denote their generation by the revolution, in accordance with some law, of a line round a fixed axis.

Perhaps more generally applicable definitions of the generation of conical and cylindrical surfaces may be thus stated :—

Def. (17). If an indefinite straight line always pass through a point and also through any curve, both point and curve being fixed in position, the straight line will generate a conical surface ; if the curve be a circle, the surface is that of the common cone ; if, further, the line passing from the vertex or fixed point to the centre of the circle be perpendicular to the plane of the circle, the surface generated is that of the right cone.

Note.—Two cones are generated, being opposite in direction, and having the fixed point as a common vertex ; these opposing cones are sometimes called "sheets."

Def. (18). The line generating the surface is called the generatrix.

Def. (19). If an indefinite straight line always pass through any curve, and also move parallel to a fixed straight line, it will generate a cylindrical surface. The common and right cylinder are special cases of this solid, analogous to the common and right cone respectively.

Note.—Surfaces are represented in drawings by the projections of their generatrices in successive positions. As most surfaces can be generated in more than one way, that form of generatrix, and law of its motion, which admit of easy delineation should be selected by the draughtsman.

In describing the regular solids it was assumed that in each case they rested on a face on the horizontal plane.

But other positions are possible. The conditions of position are usually satisfied by one of the cases following :—

Case (1). One face in or parallel to the horizontal plane.

Case (2). One edge in or parallel to the horizontal plane, and the inclination of one of the planes containing that edge given.

Case (3). The inclinations of one face and of a line lying in that face given.

Case (4). The inclinations of two edges or diagonals given.

Case (5). The inclinations of two faces given.

Note.—Problems dealing with all these cases will be explained, but the student at first should confine himself to cases (1) and (2).

Under case (1), inasmuch as the face of the solid rests on the horizontal plane, it follows that this face will in plan be seen of the real dimensions, and so can be drawn at once. With the aid of auxiliary elevations on vertical planes, conveniently disposed, the plan of the rest of the solid can usually be drawn without difficulty.

Note.—For example, if it be required to draw the plan of a right pyramid, of given dimensions, resting on one of its faces (the base not being considered one of the faces) on the horizontal plane, it will be useful to begin with an isosceles triangle composed of one of the edges of the base and two other lines of equal length, and each corresponding to one of the slant edges of the solid. The next step might be to draw, at right angles to the prolongation of the edge of the base, the xy line of a vertical plane of projection, and to obtain an auxiliary elevation of the pyramid. From this elevation plans of the other points of the pyramid can be projected into the positions proper to them in connection with the plan of the face already drawn, and the projection completed by joining these points in a manner and order similar to those which exist in the original solid.

Under case (2) similarly it will be useful to commence with that edge which is in the horizontal plane, and to construct or build the plan of the whole solid on this line by the aid of one or more auxiliary elevations. Or, in some cases, it may be easier in working to commence with an elevation, and from that projection to derive the plan.

Note.—For example, let it be required to draw the plan of a given right prism, resting on one of the edges of its base, in the horizontal plane, and with either the base or the face containing that edge inclined at θ. Here commence with that elevation which will the most readily supply the information required for the construction of the figure, *i.e.*, the side view, or a projection on a vertical plane whose horizontal trace is at right angles to the edge on which it rests. Complete the solid in elevation, and draw the projectors for the plan. Anywhere that happens to be convenient, assume in that projector which refers to the edge on which the solid is resting a line equal to this edge, and round that build the plan of one end in accordance with the conditions of its form. Similarly, the other end of the solid can be obtained in plan, and the two opposite ends of the solid being present in plan, the solid can be completed by joining points in similar positions at each end of the solid.

Note.—Some difficulty is often felt in deciding which of the lines of a projection, representing the edges of a solid, should be represented by firm lines, and which by dotted lines implying that the latter represent unseen edges of the solid. The difficulty disappears if the draughtsman bears in mind that in projection the solid is always supposed to be between the observer and the plane on which the projection is made. And this is the case even when the *xy* line has been drawn on the paper between the already existing plan of the solid and the position of the draughtsman as he sits or stands at his drawing, for the assumption in this case is that the projection was made in the ordinary way on a vertical plane *beyond* the solid, and that for convenience sake the plane of projection was moved to some more convenient position between the position of the draughtsman and the plan of the solid, and, being placed parallel to the original position of the plane, is then revolved, carrying the projection with it on to the surface of the paper.

Sections of Solids by Planes.

We now arrive at the consideration of sections of solids by planes.

The planes employed to cut solids may be horizontal, vertical, or inclined. Every secant plane will divide the solid into two portions, and the intersections of the plane with those surfaces of the solids which are cut by it will form a plane figure which is called the "section" of the solid. [Def. (20)]

Note.—The form of the section will depend on that of the solid, and also upon the relative positions of the solid and the secant plane. The case of a right cone may be cited, the resulting forms of the sections of which, when cut by different planes, are five in number. When the cone is cut by a plane passing through the vertex the section is a triangle. When cut by a plane at right angles to the axis, the section is a circle. When cut by a plane inclined to the axis at the same angle that the generatrix makes with the axis, the section is a parabola. When cut by a plane inclined to the axis at a less angle than the generatrix makes with the axis, the section is a hyperbola. When cut by a plane inclined to the axis at a greater angle than the generatrix makes with the axis, the section is an ellipse.

Def. (21). A sectional plan or elevation of a solid is that particular representation of the solid, when cut by a plane, which shows the true form of the section, together with the plan or elevation of that part of the divided solid which lies on the further side of the secant plane, considered with reference to the supposed position of the observer.

Note.—The true form of the section made by a vertical plane is only given by a projection on the secant plane, or on a plane parallel to it. A projection on a plane which is not parallel to the secant plane may be distinguished from a sectional elevation by being called "an elevation showing a section." In making a sectional elevation of a solid cut by a vertical plane, it is usual, for the sake of clearness, to project the sectional elevation on to a plane parallel to the secant plane. To determine such an elevation it is necessary to have a figured plan, or a plan and elevation of the solid. The sectional plan of the solid, when the result of a section by a horizontal plane, always shows the true form of the section. In this case the trace (vertical) of the horizontal plane should be drawn, and the intersections of this line with those of the elevation of the solid, being projected on to the plans of these lines, the plan of the section is the result.

Def. (22). When a succession of vertically equidistant horizontal planes are used to give successive sectional plans at different levels, the solid is said to be contoured. (*See ante*, Def. (15), p. 17.)

When an inclined plane cuts a solid it will be convenient to draw an elevation of the solid, and of the plane, on a vertical plane at right angles to the horizontal trace (or the horizontals) of the plane, and from this elevation to project on to the plan the points of section made by the plane.

Note.—The projection of solids under cases (3), (4), and (5) (*see ante*, p. 26), will be considered later on.

Interpenetration of Solids.

It is sometimes necessary—as, for example, in drawing the plan of two arched passages mutually intersecting—to represent the intersections of solids one with another.

A general method for determining such intersections is to contour both or all the solids, and, since the contours of the same level on each solid must intersect (if they intersect at all within the limits of the surfaces of the solids), in points which are on the intersections of the solids, the representation of these intersections can be obtained to any required degree of accuracy.

Note.—Solids contained by curved surfaces, as well as those comprised within plane surfaces, can be treated in this way. Investigations of the intersections of curved surfaces are, however, often simplified by using a series of secant planes which are not horizontal. In fact, the principle is to select such a system of planes as will give intersections on each solid which are easily determined and entail the least labour, and give the greatest possible accuracy in delineation. Thus horizontal planes might be used for two interpenetrating cylinders whose axes are horizontal, or for a cone with its axis vertical, piercing or passing through a cylinder whose axis is horizontal. Again, for interpenetrating cylinders whose axes are inclined and do not meet, a succession of planes parallel to both axes might be used. If the two axes intersect, a succession of spheres having as common centre the point of intersection of the axes might be employed.

Note.—But all such operations require a considerable amount of careful and accurate draughtsmanship to give satisfactory results.

Solid Geometry Problems.

(INDEX SYSTEM OF PROJECTION.)

Note.—For the simpler problems it has not been considered necessary to give a figure in illustration of the steps of construction; the student is recommended to draw the figures for every problem, even in the cases where they are given in the text. They can with advantage be drawn in each case more than once, different conditions and data being assumed. When figures are given in the text they have been drawn to a scale of 40 units to an inch. In the figures accompanying the problems the following convention has been adopted: Given lines in plan are shown by continuous lines, as also are horizontal traces of planes; projectors and lines of construction are dotted; elevations of lines and planes are continuous lines; results are shown in thick lines; contours are shown chain-dotted.

PROB. I.—In a given plane, and from some point in that plane, to draw a line which shall have a proposed inclination.

Draw the plan of the plane, representing it by means of a scale, and at least two of its horizontals, one of which should be taken as passing through the point whence the line is to be drawn. Take the difference of the indices of these two horizontals, and construct a right-angled triangle whose perpendicular is equal in units of length (taken from the scale of the drawing) to this difference, and whose angle, subtending the perpendicular, is equal to the inclination of the line. The base of this triangle will evidently be the plan length of a finite line for a difference of vertical level between its extremities equal to the difference of level between the two horizontals at first taken on the plane. With the length of the base from the plan of the point (in one horizontal) describe an arc cutting the other horizontal, and join the points so obtained with the plan of the given point. These lines will satisfy the conditions of the problem.

Note.—It is not necessary to draw the triangle of inclination for the line. The plan length of line may be calculated by means of a table of cotangents.

The arc, that has been used from the point assumed or given in one horizontal, to cut the other horizontal of the plane, may either cut it in two points, be tangent to it, or not cut it at all. In the first case, two positions of the line are possible; in the second, only one (and in this case the line has the same inclination as the plane); and there is no solution possible in the third case, as the inclination of the line is greater than that of the plane.

PROB. II. (Fig. 10).—To draw through a given line a plane of given inclination. (This problem is the converse of the last.)

Let a_5b_{40} (Fig. 10) be the plan of the given line. Let the angle of inclination of plane be (α). In line (a b) take any two points of convenient indices (in this instance say the points whose indices are 20 and 30). Draw also, anywhere that is convenient, an elevation of a cone (or section through the axis on a vertical plane) whose generatrices are inclined to the base at the same angle as the inclination of the plane to the horizon, and whose axis is, in length of units of the scale of drawing, the same as the difference (10) between the indices of the points in the line figured 20 and 30 respectively. Suppose this cone placed in connection with the line in such a manner that its vertex coincides with one of the points (20 or 30) in the line, and the base is horizontal. When this is the case it is evident that the horizontal base, being 10 units below the vertex, is in the same horizontal plane as the other point (30 or 20). A line from this last point tangent to the projection of the base of the cone (which is a circle), will be one horizontal of the plane required, and a line, parallel to this line, through the vertex of the cone (20 or 30), will be another, and both these will be horizontals of a plane, which touches the surface of the cone, and which will therefore have the required inclination (α).

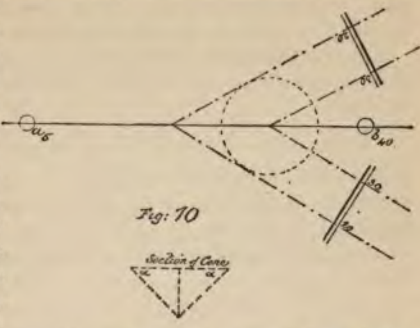


Fig: 10

Note.—As there were three cases in Prob. I., so there are three cases in this problem. If (α) be less than inclination of line the problem is impossible. If (α) be equal there is one plane possible, and its contours will be at right angles to plan of line. If (α) be greater than inclination of line two planes, one on each side of line, are possible. The line, in fact, is the intersection of the two planes.

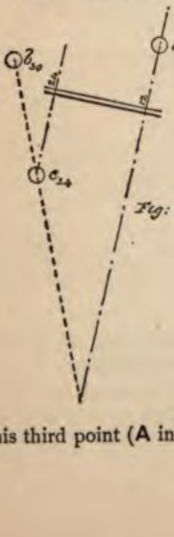


Fig: 11

PROB. III. (Fig. 11).—To determine the scale of slope of a plane which is to contain three given points.

Let $a_{12}b_{30}c_{24}$ (Fig. 11) be the figured plans of the three points. Join any two of them by a straight line, and find on this line a point which shall be at the same level (and therefore carry the same index) as the third point. If this third point (**A** in Fig. 1) be joined to the point fixed on line **BC** by a straight line, this line will evidently be one horizontal of the required plane, and another horizontal drawn through either of the points (say c_{24}) will give us the means of graduating the scale of slope.

Note.—The three points must not in plan be given in one and the same straight line, as the only plane containing them will in such a case be vertical.

PROB. IV.—Through a given point in space to draw a plane parallel to a given plane.

As the planes are to be parallel, the scale of slope of the required plane can at once be drawn where it is convenient to place it, and parallel to the scale of slope of the given plane. By projecting a horizontal from the given point on to this scale, the index of one horizontal is at once figured on the scale, inasmuch as the point is to be in the required plane. The scale of slope may now, starting from this fixed index, be further divided similarly to the division of the scale of slope of the given plane.

Note, and Axiom (35).—The conditions for the parallelism of two planes are :—(*a*) Their horizontals (and, therefore, their scales) must be parallel. (*b*) For equal differences of level there should be measurable equal lengths on both scales. (*c*) The planes should both proceed in the same direction, *i.e.*, they should both rise or fall together.

PROB. V. (Fig. 12).—To determine the intersection of two planes given by their scales of slope.

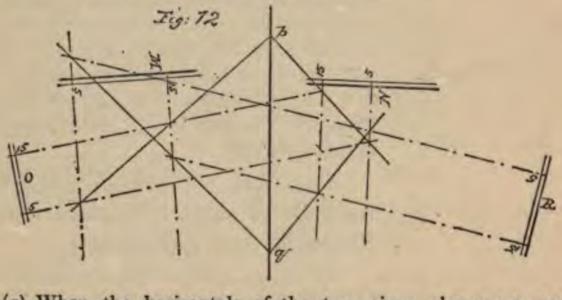


Fig: 12

(*a*) When the horizontals of the two given planes are parallel (although the planes are not therefore necessarily parallel (*see* Note to Prob. IV.), the elevation of the intersection can be found by making elevations of the planes on a common *xy* line. The projection of this intersection on the horizontal plane of projection will be a line parallel to the contours of the two planes. It can be figured from the elevation.

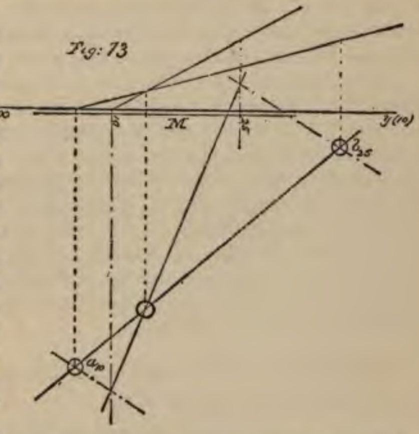
(*b*) When the horizontals of the given planes intersect at a convenient angle, it is only necessary to take similarly figured horizontals of each of the planes, and to find their intersections. The points thus obtained are plans of points in the intersection of the two planes, and are figured by the horizontals determining them.

Note.—It is only absolutely necessary to find two such points.

(*c*) When the horizontals of the given planes intersect when produced (if necessary) at an inconvenient angle. Let **MN** be the scales of slope of the two planes (Fig. 12). Assume any plane **O**, figured at will, and find its intersection with each of the given planes. These intersections will meet in a point (*p*), which is one of the points in the intersection of the given planes **M** and **N.** Similarly, with another assumed plane **R** find another point (*q*). The line (*pq*) will represent a plan of part of the intersection of **M** and **N.** The line can be figured from any of the scales of slope belonging to the planes in which it lies.

PROB. VI. (Fig. 13).—To find the intersection of a line with a plane.

Let **M** be the scale of slope of the given plane, (*ab*) the plan of the given line. Assume any two points of figuring in the line, and through these draw two parallel lines, in a convenient direction, to represent the contours of a plane containing the given line. Find the intersection of the assumed and given planes. It is evident that this intersection (produced, if necessary) will meet the given line in the point in which the latter pierces, or impinges on, the given plane.



Note.—Another method. Draw an *xy* line at right angles to horizontals of the given plane and make elevations of the line and plane. From the elevation of the point of intersection project the plan of the point, which, of course, is to be found in the plan of the line.

PROB. VII. (Fig. 14).—To measure the angle contained by two lines.

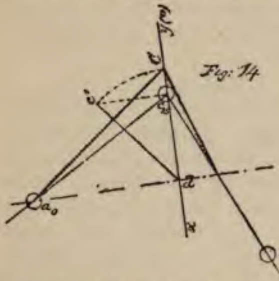


Fig: 14

Let (*ac*), (*bc*) be the plans of two lines, (*a***C**), (*b***C**), meeting at **C** (plan *c*). Determine any convenient horizontal of the plane containing the two lines (Prob. III.). If, on this horizontal as an axis, the plane is revolved till it coincides with the horizontal plane, carrying the point **C** with it, the true angle at **C** will be shown when **C** is in the horizontal plane. From (*c*) draw (*cd*) perpendicular to the horizontal used as an axis, and on this line, used as an *xy* line, make an elevation of (*d***C**). By cutting (*dc*) produced with the real length (*dc'*) of the line from (*d*) to **C**, the point **C** is given on the horizontal plane; for, in revolving round the axis, the point **C** will move in a vertical plane, whose trace is (*dc*) and will be found in (*dc*) produced. By joining the point **C** with those points in which the given lines are intersected by the horizontal axis, the real value of the angle at **C** is given, and can be measured.

Note, and Def. (23).—This plan of bringing an inclined plane into coincidence with the horizontal plane (or, as it is technically called, "construction"), with the object of obtaining information respecting the *real* relation of lines to one another, is of great use. By its use the true form of sections of solids made by inclined planes, and shown on figured plans, can be deduced. In the construction of models of solids composed of a combination of planes, such as are those of earthworks employed in fortification, the true form of any portions of the surface of the model can be ascertained by constructing the planes of which they are parts. These true forms can, when obtained, be pieced together to form the model solid.

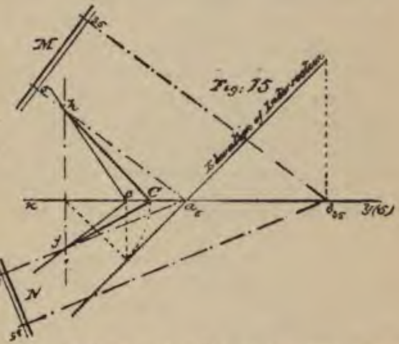
PROB. VIII.—Through a given point to draw a line perpendicular to a given plane.

A line drawn through the plan of the point perpendicular to the horizontals of the plane will be the plan of a line impinging on the plane at right angles to its surface. To find the plan of the point of intersection of the line and plane, make an elevation of the line and plane and obtain the elevation of the point of intersection, and project from this point on to the plan of the line.

Prob. IX.—Through a given point to draw a plane perpendicular to a given line. (This problem requires no explanation, being the converse of the last.)

To figure the scale of slope of the plane an elevation will be required, the *xy* line being taken parallel to the plan of the given line.

Prob. X. (Fig. 15).—To measure the dihedral angle enclosed between two planes.



By taking an auxiliary plane, perpendicular to the intersection of the given planes (Prob. IX.), and finding its intersection with both of the given planes, we get the projection of the angle contained by the given planes. This angle being "constructed," the true value is obtained, and can be measured. Let **M**, **N** be the scales of slope ot the two given planes, (*ab*) the plan of their intersection. Draw any horizontal (5) of the auxiliary plane at right angles to (*ab*), and find the intersection (*c*) of (*ab*), and the auxiliary plane (Prob. VI.), and also the intersections (*ch*) and (*cj*) of the auxiliary plane with the given planes **M** and **N** (Prob. V.). The angle (*hcj*) is the projection of the angle contained by the two given planes, and this being constructed, as shown in figure, the angle (*h*C*j*) is the true value of the angle.

Note.—This problem can be used to draw a plane through a line lying in another plane, so as to make some proposed angle with that plane.

Prob. XI.—From a given point to draw a line to meet two given lines which are not parallel.

Through the given point and each given line draw a plane (Prob. III.), and find their intersection (Prob. V.). This is the required line, which must evidently (produced if necessary) meet the two given lines.

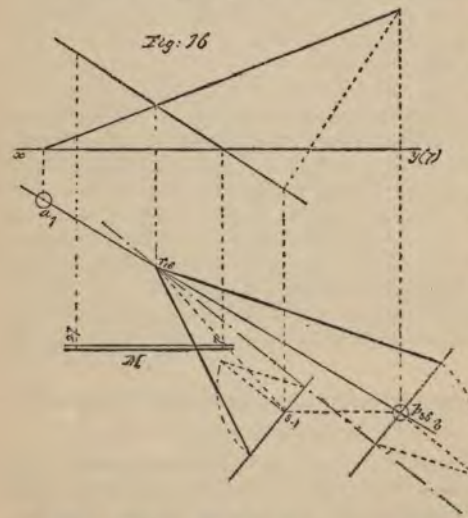
Prob. XII.—To draw a plane to contain one given line and to be parallel to another given line.

From any point, in that given line which is to be contained by the plane, draw a line parallel to the other given line. Pass a plane through these two lines (Prob. III.). This plane evidently satisfies the conditions.

PROB. XIII.—To draw a line perpendicular to two given lines, which are not parallel, and which do not meet.

Determine (Prob. XII.) a plane to contain one of the given lines and to be parallel to the other. Draw from any point in the latter line a perpendicular to the plane (Prob. VIII.), and determine the point where it meets the plane (Prob. VI.). From this latter point draw, in the plane, a line parallel to the second line. This line being in the same plane as the first line, must meet that line, inasmuch as it is not parallel (by the conditions) to it. From the point of meeting draw a line at right angles to the plane. This is the perpendicular required.

PROB. XIV. (Fig. 16).—To determine and measure the angle made by a given line with a given plane.



Let **M** be scale of slope of given plane, and (*ab*) the plan of the given line. On any convenient *xy* line make an elevation of the plane and of the line. Find (Prob. VI.) the intersection (*r*) of the line and plane. Assume any point (*p*) in the line, and draw a perpendicular to the plane from this point (Prob. VIII.). Find the intersection (*s*) of this perpendicular with the plane, and join (*rs*). Then in figure (*prs*) the angle (*prs*) is the angle required, and being "constructed" its true value can be ascertained.

PROB. XV. (Fig. 17).—To draw the plan of any plane polygon when its plane is inclined at a given angle.

Let **ABCDE** be the polygon, and (*a*) the inclination of its plane. Take any *xy* line, and draw the elevation of the plane and also its horizontal trace (at right angles to *xy*). The polygon can now be revolved into the plane, as shown in the figure, on the axis of the horizontal trace, until it is brought into the elevation of the plane, whence it can be projected in the usual manner on to the horizontal plane.

Note, and Def. (24). — This operation, which may be called "Re-construction," is the converse of that in Prob. VII. Each point of the plan, (*a*), (*b*), (*c*), &c., will be found in lines drawn parallel to the *xy* line, and passing through the original points **A, B, C,** &c.

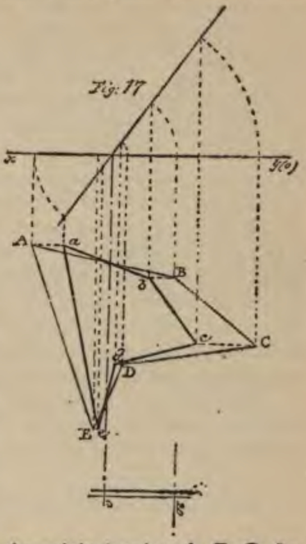


Fig: 17

PROB. XVI. (Fig. 18).—To draw the plan of any plane polygon when two of its adjacent sides are inclined at given angles.

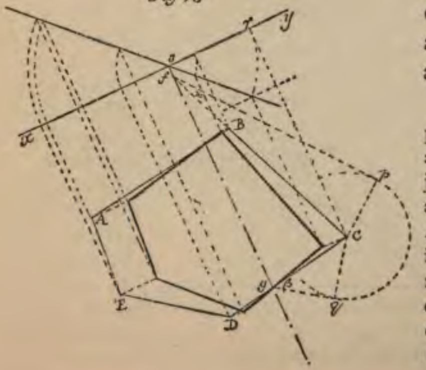


Fig 18

Let **ABCDE** be the polygon, (*α*) and (*β*) the angles at which the adjacent sides **CB, CD,** are respectively inclined. At any convenient point in **CB**, or **CB** produced, make the angle (**C** *fp*) equal to (*α*), and from **C** draw the perpendicular (**C***p*) to the line (*fp*).

With centre **C** and radius (**C**p) describe a circle, and to this circle draw a tangent, on the side of **DC**, making with **CD** an angle equal to (β), and meeting this line in the point (g). From **C** draw (**C** q) perpendicular to (gq). Then (fp) and (gq) are the plans of (**C**f) and (**C**g) when the figure **ABCDE** is fixed in space according to the conditions, and the line joining (f) and (g) is one of the horizontals of the plane in which the polygon lies when it fulfils the conditions of the problem. From (**C**) draw a projector (**C** r) perpendicular to an xy line, assumed where convenient, and taken at the level of the horizontal of the plane of the figure just determined, and draw the elevation of the plane by drawing parallel to the xy line a line at the distance (**C**p), and intersecting this line with an arc struck from centre (o), and radius equal to (or). The point so obtained, and joined with the centre whence the arc was struck, will give the elevation, or vertical trace, of the plane. This point will also be the elevation of the point **C** when it lies on the plane of the polygon, and the rest of the plan may be completed by the methods described in Prob. XV.

Note.—If (α) + (β) + angle **BCD** = 180° the figure would be in a vertical plane, and, therefore, the sum of the angles of inclination, and the angle contained by the two lines concerned, cannot exceed 180°.

PROB. XVII. (Fig. 19).—Through a given line to draw a plane to make an angle (a) with a given plane.

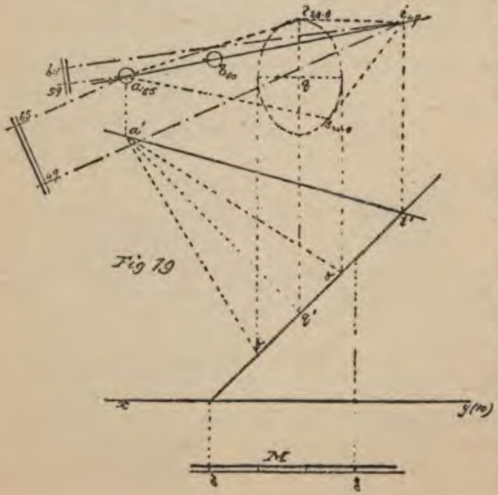


Fig 19

Let (ab) be plan of line, **M** scale of slope of given plane. Find, in the first place, the point in which the given line intersects the given plane (Prob. VI.). Let (i) be the plan of the point of intersection. From any point (a) on the line draw a perpendicular to the plane (Prob. VIII.), and find point of intersection (q) of this perpendicular with the plane. With vertex (a) and axis (aq) construct a right cone with slant side inclination equal to (α). Determine and draw the ellipse corresponding to its base when standing on the plane **M**. From (i) draw tangents to the ellipse (it), (is). Then the planes containing the points whose plans are (i), (a), (s), and (i), (a), (t), will evidently satisfy the required conditions. The scale of slopes of these planes can be drawn by the use of Prob. III.

Note.—(α) must not be less than the angle made by the given line with the given plane.

Note.—The tangents to the ellipse may be more correctly drawn by "constructing" the ellipse and the point (i), and finding the points of contact on the circle, which can then be again turned up into the plane.

PROB. XVIII. (Fig. 20).—Through a given point to draw a plane of given inclination to make a required angle with a given plane.

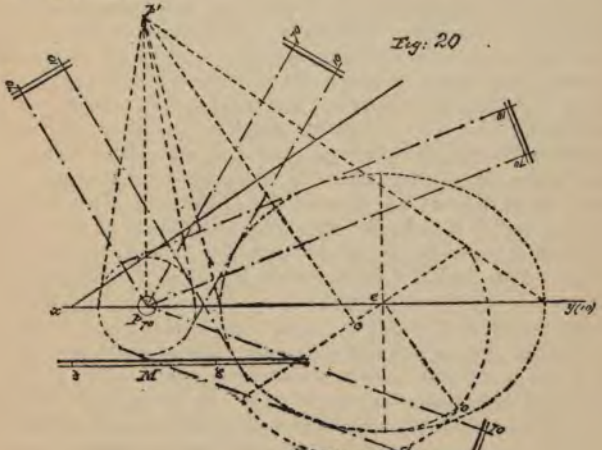


Fig: 20

Let **M** be scale of a slope of given plane and (p_{70}) the plan of the given point. Take (p) as the common vertex of two cones, one with slant side at the same inclination as the required plane, and the other with slant side making with the plane **M** the required angle. The cones to be both right ones: the axes being respectively vertical and perpendicular to the plane **M.** A plane (or planes) tangent to the surfaces of both cones will satisfy the conditions of the problem. On any convenient level determine the horizontal traces of both cones; in the case of the cone with vertical axis this will be a circle; in the case of the other it will be an ellipse, or some other curve (*see* p. 28). Lines tangent to both these projections will, in each case, be the horizontal trace of a plane which will touch the surfaces of both cones, and pass through the point (*p*), the index of which can be used to graduate the scale of slope.

Note.—If the horizontal projections of the cones touch each other externally, there are three planes that will satisfy the conditions; if internally, there is only one plane; if they intersect, there are two planes; if one projection fall wholly within the other, there is no plane, or the problem is impossible.

Note.—By reference to Fig. 20 it will be seen that the major axis of the ellipse is given by the production of those generatrices of the cone which are bounding lines of the section of the cone made by a vertical plane passing through the axis. The major axis being bisected, the centre of the ellipse (*e*) is obtained, and the direction of the minor axis being at right angles to the major axis, can be drawn; it is necessary, however, to determine its length. It will be seen that a line joining the centre of the ellipse to the vertex does not, however, coincide with the axis of the cone. To obtain the length of the minor axis, take a section of the cone, at right angles to its axis, and passing through the point where the line drawn from the vertex to the centre of the ellipse meets the horizontal plane on which the ellipse lies. This section of the cone will be a circle, and the intersection of the plane of this circle with that of the ellipse will be a horizontal line through the centre of the ellipse. That portion of this line which lies within the circumferences of the circle and the ellipse (which evidently intersect on this line) will be the minor axis of the ellipse. Let (*e*) be centre of ellipse, ($p'c$) the axis of the cone. Through (*e*) draw a line perpendicular to ($p'c$), meeting ($p'c$) in (*c*), the centre of the circular section of the cone, made by a plane, of which (*ec*) is a portion of the vertical trace. On (*ec*) produced both ways describe, with centre (*c*), a circle (or semi-circle) representing the whole (or part) of the circular section of the cone; then the ordinate (*eo*) is the length of the semi-minor axis of the ellipse, whose centre is (*e*).

In treating of the Projection of Solids it was pointed out that there were five cases to which the data regarding position could usually be referred (*see* p. 26). To illustrate the methods of projection only the two first cases of position were then examined, and the following problems will dispose of the other three cases: These are—

3. When the inclination of a face and ot a line in that ace are given.

4. When the inclinations of two edges or of two diagonals in a face are given.

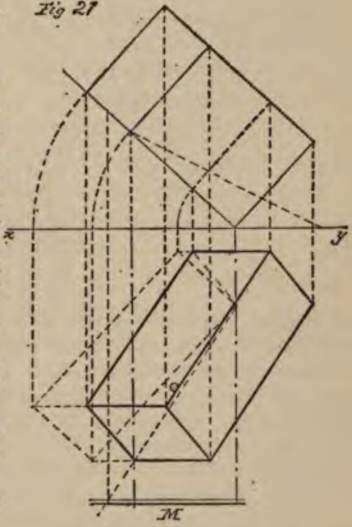
5. When the inclinations of two faces are given.

PROB. XIX. (Fig. 21). Given the inclination of one face of a solid and of a line in this face to determine the plan of the solid.

Draw the scale ot slope **M** of the given plane. Place in this plane a line inclined at the inclination (of the line) given (Prob. I.). "Construct" this line (Prob. VII.). On the constructed line draw the true form of that face of the solid which is nclined at the given angle of the plane **M**. "Re-construct" this polygon into the plane **M** (Prob. XV.), and obtain its plan. Make an elevation of the plane **M** and of the polygon, and complete the elevation of the whole solid. From this elevation can be projected the required plan.

Note.—The solid represented s a right rectangular prism.

Fig 21



PROB. XX.—Given the inclinations of two adjacent edges of a solid to draw its plan.

The preliminary steps of this problem have been explained in Prob. XVI. Having obtained the plan of the face of the solid, it is only necessary to complete the elevation, as in the case in the last problem, and to obtain the plan of the solid by projection from this elevation.

PROB. XXI. (Fig. 22).—Given the inclination of two faces of a solid to determine its plan.

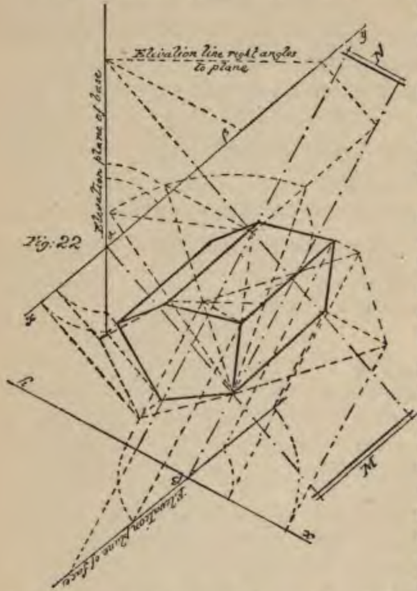


Fig: 22

Note.—The solid taken for the purposes of illustration and drawn in the figure, is a right irregular pentagonal prism.

Let **M** be scale of slope of inclination (α) of base of solid, and let (β) be inclination of one face of the solid. Draw an elevation of the plane **M**. At any convenient point in the elevation of **M** draw the elevation of a line at right angles to plane **M**, and determine its plan. Through the line pass a plane **N**, inclined at (β). Find intersection of **M** and **N**. This line will evidently be the indefinite length of an edge of the base of the solid. "Construct" the plan of this line; first as in plane of base, and draw the true form of the polygon of the base on the constructed line; secondly, as in the plane of the face, and draw, on the constructed line, the true form of the face. "Reconstruct" these figures into their respective planes, and project their plans. Complete the plan by drawing parallel lines, according to the composition of the solid, through the points already obtained.

Note.—When, by the nature of the solid, the planes of the two adjacent faces, whose inclinations form part of the data of this problem, are not at right angles to each other, the first steps of the working out will require the employment of Prob. XVIII. The point assumed in the plane **M** is taken to be the vertex of two cones (*see* Prob. XVIII.), and that one of the resulting planes selected which will be the most convenient for the purposes of representation. After the horizontals of this plane have been determined, the further working out of the problem is the same as that given above.

Problems

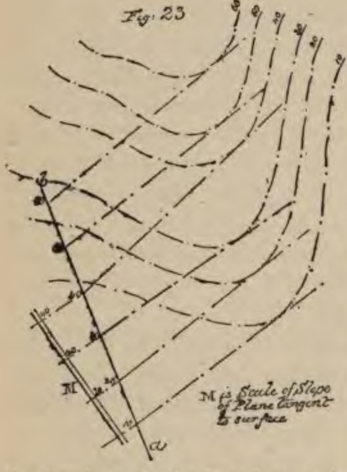
IN CONNECTION WITH IRREGULAR SURFACES, AND RELATING TO THE DEFILADE OF WORKS OF FORTIFICATION.

In drawings, prepared for the working out of designs of works of fortification represented on paper by figured plans, the ground, at the sites of, and in the neighbourhood of, the proposed works, is represented by a series of figured contours given by the intersection of horizontal planes with the surface of the ground (*see* Def. 15), the datum or original horizontal plane to which these contours are referred being taken at some convenient level. The closeness of vertical intervals at which these contours occur depends partly on the scale of the drawing and partly on the character of the ground represented. It is usually assumed that the vertical difference of level between any two successive contours has been fixed on the assumption that the surface of the ground lying between them is normally plane. The various problems connected with the planes of fire and defilade, the intersections of these planes, and of others related to them, with those of the ground and with each other, can be worked out by methods similar to those already detailed. For example, the intersections of regular planes (of which the bounding surfaces of earthworks are for the most part composed) with each other, and with the irregular planes of the ground, may be determined by the use of Prob. V.; or the representation of a road passing over the surface of ground under certain limitations of gradient may be determined by the use of Prob. I.; or the cuttings required to be made through undulating ground, in order that the fire, proceeding from some fixed battery or face of a work, may search out some portion of the foreground, unseen till those cuttings are made, are determined by the use of Probs. II. and V.

Such simple applications of the problems already given do not require any detailed explanation. But there are a few others which, being constantly required in the design or defilade of works, may now be noticed at greater length.

PROB. XXII. (Fig. 23).—To determine a plane containing a given straight line, and tangential to ground represented by contours.

Fig: 23



Let (*ab*) (Fig. 23) be the plan of the line, (*a*) being the lower point. Let the curved lines represent a contoured plan of ground. Determine in (*ab*) points of the same indices as those of the ground contours, and from each point so figured draw a line tangent to the contour of the same index. Then that one of these lines which makes with (*ab*) towards (*a*), the lower end of (*ab*), the smallest angle will be a horizontal of the plane that is tangential to the surface of the ground given in the drawing. In the first place, each of the lines drawn from (*ab*), tangential to successive contours of the ground, is one horizontal of a plane, which touches the surface of the ground at the point of contact of the line and the contour. If the relative positions of (*ab*) and the ground are such that the planes represented by each of these lines rise from (*ab*) towards the surface of the ground where contact is about to occur, each of these lines will include with (*ab*) towards the lower end of it an angle less than a right angle; if, on the other hand, the relative positions of (*ab*) and the contoured ground are such that the planes fall from (*ab*) towards the surface where contact is to occur, the angles included by these lines towards the lower end of (*ab*) will be obtuse angles. For the sake of brevity, let us call these two classes of planes "Rising" and "Falling" planes. Now, of all rising planes, passing through a line, that one is the steepest whose horizontals make the least angle with the lower part of the line. Further, within the possible limits of selection in each individual example or problem that is under consideration (and here in the case represented in the figure the range of possible planes is determined by the number of separate tangents drawn to the contours of the ground), it is evident that the steepest must determine the point of contact, for, were this not the case, any plane less inclined would cut the ground. Hence, in the case of a series of horizontals, drawn through points in the line tangent to the ground contours of similar indices, and which all represent "rising" planes, that one which makes the angle included between itself and the line, towards the lower part of it, relatively the least, determines the plane of contact. Similarly of all falling planes, passing through a line towards the surface of adjacent ground, it may be shown, subject to the limitations of the requirements of the case, that the one of least inclination will determine the point of contact. Now planes falling from a line will in every case be represented by horizontals making an obtuse angle with the lower part of the line. And, of all such planes, the less the inclination of the plane, the smaller the angle included between the horizontals and the line, toward the lower part of it. Therefore it may be stated that in the case of a series of horizontals, representing in each individual case a descending plane, drawn through points in a line tangent to contoured ground of similar figuring, that one which makes with the line towards the lower end of it the least obtuse angle will determine the plane drawn through the line, and tangent to the ground.

Note.—If the given line be horizontal, tangents to the contours of the ground should be drawn parallel to the line. That tangential horizontal which, in the case of planes ascending from the line (which itself is one horizontal of all planes containing it), gives the steepest plane: or which, in the case of descending planes, gives the plane of least inclination, fixes, in each case, the tangent plane.

Note, and Def. (25).—"Rising" and "falling" planes may be more accurately defined as follows:—A plane, passing through a line of fixed position, rises, or falls, in the direction of a given surface, according as a perpendicular to its horizontals, lying on the surface of the plane, and drawn from the lowest part of the line through which it passes, and at the same time *up* the plane, lies respectively nearer to, or farther from, the surface than the line itself.

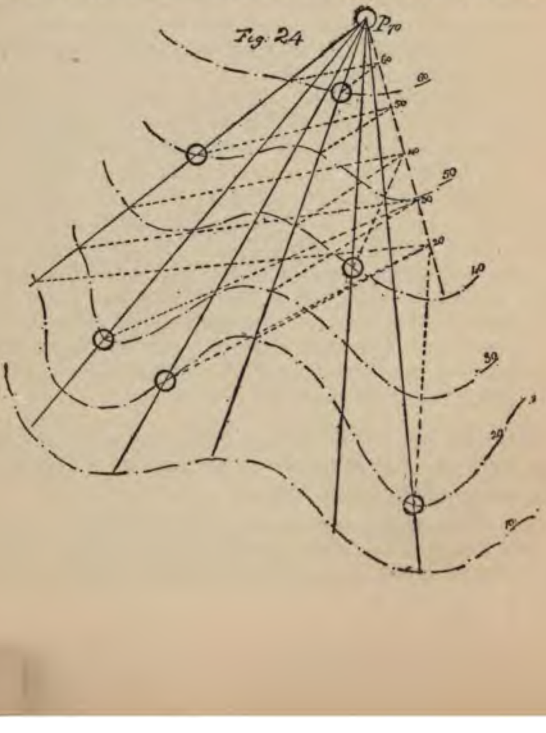
PROB. XXIII.—Through a given point to draw a plane which shall be tangent to two prominences. (This is only an extension of the last problem.)

Between the prominences, and passing through the point, place any line, and from the point graduate and figure it in a manner similar to the ground. Draw, from the points thus obtained on the line, tangents to correspondingly figured contours of the ground on each side of the line, and find the tangent plane to each prominence by the last Problem. If the resulting planes were portions of one plane surface, it is evident that the horizontals, fixing the planes of contact on each side of the line, would be parallel to each other, and the resulting scales of slope would, therefore, be parallel and equal. If this result is not obtained at the first graduation of the line, other trial lines from the given point must be drawn, or the first line drawn be re-graduated, and the whole process repeated till the horizontals, giving the planes of contact on each side of the line, become parallel, *i.e.*, till the two planes, passing through the line, and therefore the point, are portions of one and the same plane surface.

Note.—The number of trials depends so much on the results given by the first, that it is important to pay particular attention when graduating the assumed line for the first time to make as close an approximation to the final result desired as may be possible.

PROB. XXIV. (Fig. 24).—From a given point to draw lines to touch a given irregular surface, represented by contours.

Fig. 24



From the plan of the point, and embracing that portion of the surface to which consideration is limited, draw the plans of the lines the points of contact of which with the surface have to be determined. From (p), the plan of the point, draw a line in any convenient direction, and graduate this line from (p) in a manner similar to the ground. Taking the plan of each transverse line in succession, join the points where it meets each contour with the corresponding points of the assumed line. By reasoning similar to that of Prob. XXII. it may be shown that, of all the lines of junction for each transverse line, that one which makes with the lower part of the assumed line the least angle determines the point of contact of the transverse line with the surface of the ground.

Note.—Having fixed the points of contact of each transversely drawn line, the point of contact and the point of origin (p) serve to determine the graduation of the line. If the number of transverse lines drawn are sufficiently close to each other, the portion of space included between any two adjacent lines may be treated as a plane; the contours of which can be determined by using the indices of the point of origin and of the two points of contact (Prob. III.). The intersection of these planes (Prob. V.) with the ground within the limits of the two transverse lines will give a close approximation to the limit of vision, or of grazing fire, as seen, or delivered, from the point of origin.

Note.—The production of these planes in the direction of vision, or fire, will also serve to determine that portion of the surrounding surface which, immediately beyond the trace of the limits of vision, or fire, is below the planes bounded by the transverse lines tangential to the ground, and, therefore, also that portion of the ground which, owing to accidents of formation, is again brought into, or above, these planes. In this way the ground, invisible, or unsearched by fire, from a certain fixed point, may be ascertained approximately, the closeness of the approximation depending on the number of transverse lines drawn.

Note.—The information resulting from the operations described in this problem may also be obtained by drawing a section on a vertical plane, the horizontal trace of which, in the case of each transverse line, will be the plan of that line. This method, however, unless the scale of the drawing is very large, is not so accurate as the other here explained, and certainly involves, in most cases, considerably more drawing.

PROB. XXV. (Fig. 25).—Within given limits of distance to determine the most commanding point of the ground with respect to the position of a given point.

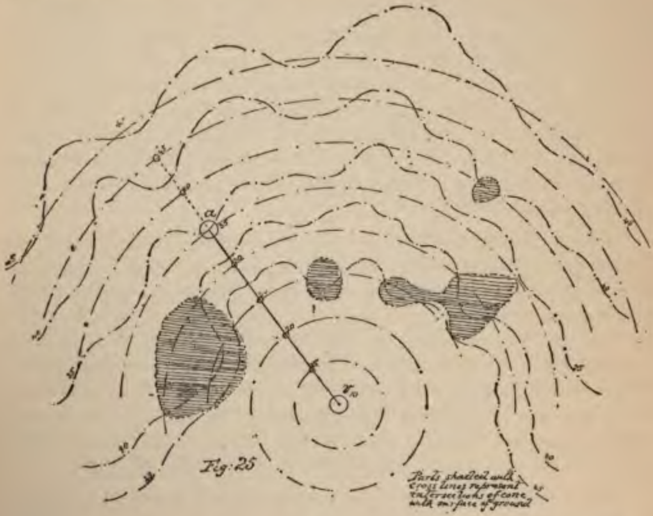

a
v_{10}
Parts shaded with cross lines represent intersections of cone with surface of ground

Fig: 25

Note, and Det. (26).—All points on the surface of ground surrounding any fixed point, when above the level of that point, are said, with respect to that point, to be "commanding." If such ground points be joined by lines to the fixed point the relative inclinations of these lines afford a measure of "command"; the greater the inclination of any one of these lines, the greater the command of the ground point to which it relates.

Assume some point (*a*) (Fig. 25) on the contoured surface of the ground which appears likely to be the point of greatest command, and join it with the fixed point (*v*). With generatrix (*va*) and vertex (*v*) describe a right cone. Graduate the line (*va*) from (*v*), in a manner similar to the ground, the indices of (*v*) and (*a*) being both given. he cone at intervals corresponding to those at which the the ground occur. Find the intersection of the surface of ith the ground. A comparison of the depths of those ground which project sufficiently to be included within cone, due regard being had to their distances from (*v*), will determine the most commanding part of the ground with respect to the vertex (v).

Note, and Def. (27).—The commanding points are also termed "Points of danger."

Note.—This problem is an example of the use of contours to find the interpenetration of solids.

Def. (28). By the term "Defilade of works" is to be understood such an arrangement of the interior of a work that it be, at the least, concealed from the view of the enemy, and also, if possible, be protected from the effects of fire.

To satisfy these requirements the crest lines of the parapets or ramparts may be raised, the terrepleins lowered, and parados and traverses constructed to cover the interior of the work from the effects of reverse, enfilade, and frontal fire. To facilitate the practical examination and determination of points of danger it is essential that accurate contoured plans of the site of a proposed work, and of the ground in the neighbourhood, should be used. On these, within the limits required, the points of danger should be determined (Prob. XXV.), and the plane, or planes, of defilade for the whole proposed work, or for the work considered in certain sections, should be drawn, and the arrangement of the interior of the work considered and designed in the manner that appears most suitable. The planes of defilade are usually assumed to pass at least the height of a man's shoulder ($4\frac{1}{2}$ feet) above the points of danger to which they refer, and the points of danger are, in consequence, commonly determined by a cone whose vertex is four and a half feet lower than the actual extent of "cover" required at any particular part of the work which is to be defiladed from those points. Using those planes of defilade which may be considered, in each case, the most convenient, it will be necessary to consider whether the raising of crest lines, the lowering of terrepleins, the construction of parados and traverses, or any combinations of these methods of defilading the whole, or various parts, of the work will be most advantageous in the particular instance under treatment. Speaking generally, any face of a work, commanded by ground in front, in rear, or in its prolongation, may be defiladed in the first case by arrangement of terreplein; in the second by the use of traverses and parados, or by placing the crest of the face in or below the plane of defilade of some other face interposing between itself and the point of danger; and in the third (unless the face under consideration is short), by the construction of traverses, which, by dividing the face into short portions, will afford it protection from the effects of enfilade fire.

Isometric Projection.

Isometric Projection, as already stated (*see* p. 6), is merely a modification of Orthographic Projection. It differs from the latter in the assumption that the plane of projection, on which the object is projected, is always, with regard to the original object, in a fixed and constant relation. This relation is one of position, and its principle may be thus stated:—

If three straight lines, meeting at a point, and being themselves mutually at right angles, make equal angles with a plane, their projections on that plane will also make equal angles with one another. Axiom (36).

Any equal lengths of these lines will also have equal projections, and hence distances, plotted on these projections, or on lines parallel to these projections, and representing projections of other original lines parallel to the three first mentioned, will bear the same relation to each other that exists between corresponding segments of the original lines. Axiom (37).

The Plane of Projection may be assumed at will either horizontal or vertical. The resulting projection might, therefore, be considered either a plan or an elevation. In general it will be more convenient to consider the projection a plan, a convention, moreover, more in harmony with that conventional modification of orthographic projection which has been called the Index system of projection.

Let a cube (Fig. 26) be so fixed in position, with regard to the horizontal plane, as to have the three edges, meeting in one of the angular points of the solid, equally inclined. Then the diagonal of the cube passing through this point will be vertical, and the planes of the faces of the cube contained by those edges of the cube will be inclined at equal angles. Complete the projection of the cube. The resulting plan is a hexagon in its bounding lines, and the plans of the three edges meeting at the point (*b*) or (*h*) make angles of 120° with each other.

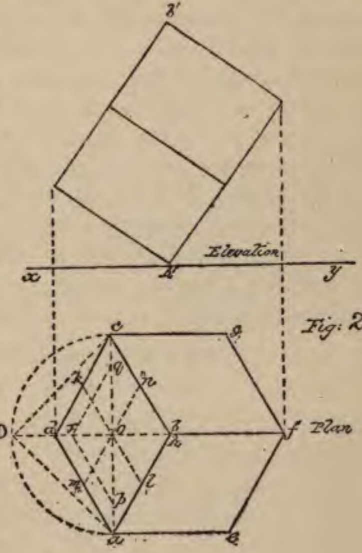


Fig: 26

Def. (29). This projection is called an "Isometric projection or view," and the directions of the plans of the three edges from the point (*b*) or (*h*) are called "Isometric axes."

Def. (30). Any lines parallel to the axes in the projection o a solid are called "Isometric lines."

Hence, all right angles in an original solid are represented, in an isometric projection, by angles of 120° or 60°. Axiom (38).

From these considerations it is evident that an Isometric Projection, while supplying in a constructively simple manner a view of an object so placed that sever surfaces are presented at once to the eye, is be to the representation of those objects whose constitution consists in the main of planes and lines at right angles to each other. It is a method of projection well fitted for the representation of details of construction, and, in some cases, for the production of general views of large works, which bear some resemblance to the perspective views commonly called bird's-eye views. On looking at Fig. 26 it will be observed in an isometric projection of the faces of any cube that one of the diagonals of each face—as (*ac*), (*cf*), (*fa*), &c.—is parallel to the plane of projection. The lengths of these lines, therefore, are the real lengths of the diagonals they represent. If a semi-circle be described on (*ac*), the sides of the original square (*c*D), (*a*D), of which the rhombus (*abcd*) is the projection, will be obtained. Then, if (*o*) be the centre of the square and rhombus, the angles (D*ao*) and (*dao*) are respectively 45° and 30°, and—

$$\begin{aligned} a\mathrm{D} : ad &= \sin 120^\circ : \sin 45^\circ \\ &= \sqrt{3} \quad : \sqrt{2} \end{aligned}$$

which gives the relation between the real length of a line and its isometric length.

This relation regulates the construction of isometric scales. It is only necessary to get two lines in this relation, on one to construct the natural scale, and on the other a proportional scale, each unit of which should bear to a unit of the original scale the ratio of $\sqrt{2} : \sqrt{3}$, and to use one for natural dimensions and the other for isometric lengths. By the use of these scales a plan and an isometric view of the same object can be simultaneously worked out on paper.

Note.—In commencing an isometric drawing it is usual to draw the three axes so that one of them shall lie up and down the paper. All dimensions of the original object, parallel in direction to the three edges represented by the isometric axes, will, in the isometric projection, be drawn parallel to one or other of the isometric axes, and their lengths can at once be taken off the isometric scales. Lines not parallel (by the constitution of the object) to the isometric axes must be fixed, as regards both direction and length, by isometric co-ordinates respectively parallel to two of the axes.

Note.—It is not uncommon to make an isometric drawing by using the natural scale isometrically. The result is an isometric drawing which, in fact, represents an object larger than it is in nature, but larger in regular proportion so far as all details of the original object are concerned.

The projection of a circle in an isometric view is evidently an ellipse, of which the major axis is equal to that diagonal of the inscribed square which, when projected isometrically, is seen at its real length. Referring to Fig. 26 it will be seen that (oc) will be the major axis of the ellipse circumscribing the rhombus ($abcd$), and that (od) will be the semi-minor axis. If the centres of the bisected sides of the rhombus be joined, as shown (kl), (mn), they will be conjugate diameters of the ellipse inscribed in the rhombus, and the major axis of this ellipse can be obtained by laying off, on each side from (o), a distance equal to half the side of the original square. Let, therefore, (op), (oq) be each the semi-major axis of the inscribed ellipse. Through (p) and (q) draw lines parallel to the sides of the rhombus (pr), (qr) meeting in (do) at (r). Then (or) is the semi-minor axis of the inscribed ellipse. Since the angle (pro) is equal to 60°—

$$\begin{aligned}(or) : (op) &= \sin 30^\circ : \sin 60^\circ \\ &= 1 : \sqrt{3}\end{aligned}$$

But in an ellipse the sum of the squares of any conjugate diameters is equal to the sum of the squares of the major and minor axes, and taking $(or) = 1$ we get—

$$\begin{aligned}[2(op)]^2 + [2(or)]^2 &= 8(om)^2 \\ 12 + 4 &= 8(om)^2 \\ 2 &= (om)^2 \\ \sqrt{2} &= (om) \\ \text{Hence } (or) : (om) : (op) &= 1 : \sqrt{2} : \sqrt{3}\end{aligned}$$

In any face, therefore, of a cube isometrically projected the following relations exist between lines parallel respectively to the sides and the two diagonals of the projected rhombus.

Major axis diagonal	:	side of rhombus	:	minor axis diagonal.
$\sqrt{3}$	:	$\sqrt{2}$	:	1

Note.—In drawing isometric circles this relation is frequently very useful. For complete circles, or large arcs, specially so; but small lengths of arcs may often be more conveniently plotted by drawing, isometrically, their chords and a certain number of ordinates.

Note.—The angle made by each of the isometric axes with the plane of projection is $\cos.^{-1} \sqrt{\frac{2}{3}}$ or 35°.

Shading.

Neither shade lines (*i.e.*, the darkening of certain lines in accordance with certain conventional assumed direction of light), nor cast shadows should be used in Geometrical Drawing. A preferable manner, for giving relief to the projection of an original object, is to use shading.

By shading is meant that conventional method of colouring, with neutral tint, the various surfaces composing an object, which will enable the observer to obtain some idea of the relative inclination of those surfaces, as well as give greater relief to the drawing. The principle on which this is effected is as follows:—

With regard to either plane of projection, that surface of the solid which is parallel to the plane receives no shade tint. Of all other surfaces the depth of the tint used in each depends on the inclination of the surface to the plane of projection, and varies directly with that inclination; the larger the angle, the deeper the tint. Thus, with regard to the horizontal plane of projection, any horizontal plane in the object (which must, of course, be of the nature of a solid in the vulgar acceptation of the word), will be shadeless, while the other limit is reached if the plane is vertical, which is represented by a jet-black line. Similarly the limits of shade, with regard to a vertical plane of projection, lie between absolute whiteness for a vertical parallel plane, and absolute darkness for a horizontal plane.

Between these limits, in both cases, the draughtsman may display ingenuity in devising a scale of shade, and art in laying in the tints. Considerable advantage and great accuracy in shading results from contouring a plan ; for the closeness of successive contours, on all the planes, varying directly with the inclinations of those planes, the plan breadths, of bands of the planes given by two successive horizontal planes, afford a ready method for constructing a scale of shade. Similarly, in the case of elevations, it is only necessary to take and represent on the elevation the outlines of two vertical sections, by planes parallel to the vertical plane of projection, to obtain a similar scale of shade for the elevation.

Note.—If the vertical distance between the two horizontal planes of section used for the plan is kept equal to the horizontal distance apart of the two section planes used for the elevation, the result will be a correspondence between the shading of the plan and the elevation, which will permit one scale of shade to be used for both.

Note.—Works of fortification should be shaded on the same principle ; but it is not unusual to use shade lines as well in the delineation of details. Shade lines are dark lines, on those edges of an object which would throw shadow, on the assumption of a conventional direction of the rays of light. They should only be placed at the intersection of an illuminated with a shadowed plane when the latter is invisible from the supposed position of the observer.

Note.—No curved outline should have a shade line.

Note.—The system of shading just explained does not give very artistic results in the case of a sphere. In shading spheres, therefore, it may be better to adopt some other conventional direction of light, and to admit the relief given by the presence of reflected light.

APPENDIX.

Hints and Suggestions for Draughtsmen.

THE instruments used by draughtsmen in the operations of Geometrical Drawing may vary in kind, according to personal preferences, or as the consequence of continued habit in usage. But whatever kinds happen to be employed should be of good quality. No good work can be done with inferior tools. Drawing pens, especially, should be of the best quality; two of these, one for coarse lines and the other for fine, are sufficient.

For ruling parallel lines a heavy rolling metal ruler is on the whole the best tool. Jointed parallel rulers are bad. For small drawings the Marquois Scales, when accurate, are often convenient, and so are small ebonite set squares.

The pencil work should always be most carefully executed. No pencil harder than HH. should be used, and F. or H. is a quality to be preferred. Before inking in the drawing the pencil work should be carefully examined, and all errors, negligences, and unnecessary lines, should be rectified, remedied, and removed.

Inking in should be done with perfectly black ink. The general character, as regards boldness of style and thickness of line, should depend partly on the size of the drawing and partly on whether the drawing is to be washed with colour. In the latter case all lines should be put in at first thin. It is better not to use, for the inking in of drawings which are to be coloured, liquid Indian ink. This ink is very liable to run, unless a considerable time for drying is allowed to elapse before colouring is attempted.

The colour used should be as pure as possible. It is, therefore, better not to use, for the best effects, moist colours, but dry cake colours. It is a good plan to rub up a considerable supply of colour, and, allowing it some little time for the coarser particles to sink to the bottom of the saucer used, to use thenceforth only the lighter solution at the top, without disturbing the dregs.

If shading is employed to give relief, use, after inking in, a general neutral grey, light in tint, to be carried over all the shaded portion of the object, and by successive operations deepen the shades where required. A good shade tint is given by Indian ink; but, perhaps, the best shading colour is lamp black. After the shading is completed, colour all over light and shade, with the local colour made to a very light tint.

The local colour used for plans and elevations should be much lighter than that used in sections of the same substance. This is on the assumption that the latter have been lately cut. Shade lines, and thick lines generally, should not be put in till all colouring is completed.

In case of accidents the best remedy is the free, but light, use of the sponge. It is wonderful what results are obtained by a judicious use of the sponge in the hands of a practical draughtsman.

The convention for lines to be used in Geometrical Drawing may be as follows :—

Given Lines—Thin continuous lines.

Elevations of Lines and Planes—Chain-dotted lines.

Contours—Blue continuous lines, or, in some cases of ground, red continuous lines.

Results—Stronger lines, &c.

Indices—In red.

Lettering—In black.

Lines of construction and auxiliary constructions—Ordinary dotted lines.

Points—Should be circumscribed by small circles.

Examples.

Unit of scale (when required) may be $\frac{1}{10}$ of an inch.

SOLID GEOMETRY PROBLEMS.

1. Determine the inclinations and true lengths of the following lines, the plans of which are each $2\frac{1}{2}$ inches long :—

(1) a_{10}, b_{-20} ; (2) c_{40}, d_{15} ; (3) e_{20}, f_{20} ; (4) g_0, h_{-30}.

2. Draw a regular hexagon of 1½-inch side, and figure its angular points consecutively, as follows :—a_0, b_3, c_{17}, d_{12}, e_9, f_7. Do any of the diagonals really intersect ?

3. A line is inclined at 50°. Draw its plan and figure in this plan points whose indices are −5, 0, 11, and 20. Draw the scale of slope of a plane perpendicular to the line at the point whose index is 11. (Prob. IX.)

4. A line is inclined at 50°. Determine a line which shall make an angle of 65° with the first line, and be itself inclined at 35°. (Prob. VII.)

5. In a plane inclined at 60°, and from a point in the plane whose index is 15, draw a line inclined at 35°. (Prob. I.)

6. The horizontals of two planes make angles in plan of 75° with each other. The planes are inclined at 35° and 50° Find the intersection of the planes, and state its inclination. (Prob. V.)

7. Measure the angle contained between the two planes mentioned in the last question. (Prob. X.)

8. Three points have plans 2, 3, and 4 inches apart. Figure the points $-5 + 5$ and $+ 10$. From any one of these points draw a line perpendicular to the line joining the other two points.

(Through the point draw a plane perpendicular to line joining the other two ; find intersection of line and plane, and join this last point with that through which the first plane was drawn.)

9. Two lines exist in space which neither meet nor are parallel to each other. Determine two parallel planes, each containing one of the lines.

(Through one line draw a plane parallel to the other line, and through the latter a plane parallel to this plane.)

10. $\mathbf{A}_5\mathbf{B}_{10}\mathbf{C}_{25}$ form in plan an equilateral triangle of 3-inch side. Draw the plane containing these three points, and through **BC** draw two planes inclined at 60°. (Probs. III. and II.)

11. $\mathbf{A}_5\mathbf{B}_{10}\mathbf{C}_{15}\mathbf{D}_{11}\mathbf{E}_{14}\mathbf{F}_{17}$ (following in the order of a clock dial) form in plan a regular hexagon of 2-inch side. Determine the intersection of the planes respectively containing **ABC** and **DEF.** (Probs. III. and V.)

12. In the figure given in the last question mark the centre

of the hexagon *g*, and through *d* draw the line *gdh*, making its length equal to three inches. Index $h_{35\cdot5}$, and find where *gh* meets the planes drawn in the last question. (Prob. VI.)

13. Draw an equilateral triangle of 3-inch side. Figure the angles $a_{11}b_{16}c_{21}$. Take *ac* as the scale of slope of a plane, and *ab* as a straight line, and measure the angle between the line and the plane. (Prob. XIV.)

14. Draw a regular hexagon of 2-inch side, and figure the corners consecutively $a_{20}b_{10}c_{-5}d_5e_5f_0$. Measure the angle contained by the planes in which the points **AFD** and **BEC** respectively lie. (Probs. III. and X.)

Do the straight lines **AC** and **EB** intersect?

15. Two adjacent sides of a square (2-inch side) are inclined at 30° and 50°. Draw a plan of the square. (Prob. XVI.)

16. Draw the horizontals of a plane inclined at 65°. In this plane place a line inclined at 30°. Through this line draw another plane inclined at 50°. (Probs. I. and II.)

17. The four points $a_0b_{45}c_{20}d_{10}$ are situated at the angles of a square whose side is 2 inches. Draw a straight line that will be perpendicular to both **AC** and **BD**. (Prob. XIII.)

18. The scales of slope of two planes include an angle of 60°, the planes are inclined in the same direction at 30° and 60°. Find the intersection of the planes, and state its inclination. (Prob. V.)

19. The plan of a point a_{47} lies visually in horizontal 20 of a plane inclined at 45°. Draw a straight line parallel to this plane, containing the point, and inclined at 20°. (Prob. I.)

20. Draw the plans of two straight lines $a_{10}b_{30}$, $c_{15}d_{25}$, 2 and 3 inches long respectively, bisecting each other at right angles at *e*. From **E** draw a perpendicular **EF** (2½ inches long), to the plane containing **AB, CD.** (Prob. VIII.)

21. In the last question determine the true angle contained by **AB, CD.** (Prob. VII.)

22. Also draw the locus of intersection with the plane containing **AB, CD** of all lines passing through **F**, and making an angle of 60° with that plane. (Prob. XVIII.)

23. Two adjacent sides, each two inches long, of a regular octagon, are inclined at 10° and 30°. Draw a plan of the octagon. (Prob. XVI.)

24. Draw a square of 2 inches side, and figure its angles taken successively $a_0b_{40}c_{20}d_{30}$. Determine the angle between the planes containing **BAD** and **BCD**, and that contained by **AD** and the plane **ABC**. (Probs. III. and X.)

25. The horizontals of two planes inclined at 25° and 50° respectively, intersect at right angles. Find the angle between the planes. Through the line of intersection of the two planes draw a third plane making an angle of 40° with that inclined at 25°. (Prob. X.)

26. An irregular pentagon **ABCDE**, of dimensions given below, has its side **AB** inclined 40° and **CB** inclined 15°. Draw its plan.

AB = 15 units Angle **A** = 105°.
BC = 15 " " **B** = 85°.
CD = 15 " " **C** = 135°.
" **D** = 70°.
(Prob. XVI.)

27. Draw an equilateral triangle with a side of three inches; number its angular points $a_{20}b_{35}c_{40}$, and the central point of the triangle d_{10}. From *d* draw a perpendicular to the plane containing *abc*, and find its intersection *e* with the plane. Determine the true angle contained by **AB** and **AC**. (Probs. VIII., VI., and X.)

28. Number the six angles of a regular hexagon taken successively $a_0b_{10}c_{30}d_{40}e_{10}f_{20}$. Determine the scales of the planes containing *abc* and *def*. (Prob. III.)

29. Find in last question the intersection of *ac* with plane *def*. (Prob. VI.) Also through *d* draw a plane parallel to *abc*. (Prob. IV.)

30. The lines joining three points which are respectively 10, 15, and 25 units above the horizontal plane, form in plan an equilateral triangle of 2 inches side. Determine the real form of the triangle. (Prob. XV.)

31. In a plane inclined at 45° place a line inclined at 30°, and through this draw a plane inclined at 110° to the other plane. (Probs. I. and X.)

32. Draw **AB**, **AC** of any convenient length, meeting at **A** at an angle of 60°. Along **AB** set off the points a_5, b_{20} at 1 inch and 4 inches respectively from **A**. Let **AC** represent the scale

of slope of a plane inclined at 60° (giving **A** the index 60) falling towards **C.** Measure the angle which a_5b_{20} makes with this plane. (Prob. XIV.)

33. Draw the shortest possible line between two given lines **AB** and **CD**, whose plans $a_{20}b_{30}$ and $c_{15}d_0$ mutually intersect at 120°, and are respectively 4 inches and 3 inches long. Determine the length of this line. (Prob. XIII.)

34. A regular hexagon of 1½ inches side has a diagonal and an adjacent side inclined respectively at 30° and 50°. Draw its plan, and explain the process of obtaining it. What is the inclination of the plane in which the hexagon lies? (Prob. XVI.)

35. Three lines mutually at right angles meet at a point. The inclinations of two of them are respectively 32° and 42°. Determine and figure their plans, assuming the index of the point in which they meet to be 20.

(Determine the plans of the two lines whose inclinations are given by Prob. VII., and from the point 20 draw a perpendicular to the plane containing the two lines. This perpendicular will be the third line.)

36. Three planes mutually at right angles meet at a point figured 20. Two of these have inclinations of 30° and 58° Determine their scales of slope, and the inclination of the third plane.

(Draw one of the planes of given inclination through the point 20. From this point draw a perpendicular to the plane. Draw through this perpendicular the other plane of given inclination. Find the intersection of the two planes. Through the point 20 draw the third plane perpendicular to this intersection, and measure its inclination.)

37. Determine the sphere on whose surface lie the four points whose indices are 20, 5, 28, and 12, and whose plans are at the angles of an irregular four-sided figure, one of whose diagonals is two inches in length, and whose two faces on each side of this diagonal make equal angles of 35° and 45° with it.

(Determine the index of the centre of the sphere. Determine three planes at right angles to the middle points of any three of the lines joining the points in succession, *i.e.*, to the sides of the irregular four-sided figure. The centre of the sphere is the point in which these three planes intersect, and the radius of the sphere is the line joining the centre with any one of the given points.)

Projections, Sections, and Interpenetrations of Solids.

1. Draw the plan of a regular tetrahedron, of which each edge is 2½ inches long, when one face is inclined at 35°, and an edge in that face at 20°. (Case 3.)

2. Draw the plan of a right cylinder, the radius of its base being 1·2 inches and its axis 3·3 inches long, when its curved surface rests on the horizontal plane. Draw a sectional elevation of the solid on a vertical plane, passing through the middle of the axis, and making with its plan an angle of 50°. (Case 1.)

3. The base of a right prism 3 inches high is a rhombus whose sides are 2½ inches long, and two of whose angles are 45°. Draw a plan of the solid when the sides containing one of the acute angles are inclined respectively at 60° and 30°. (Case 4.)

4. Draw the plan of a double pentagonal right pyramid, resting with one face on the horizontal plane. Side of base, 1 inch; height of pyramids, 2½ and 1½ inches. Draw an elevation of the solid on a vertical plane, making an angle of 50° with the plan of the axis. (Case 1.)

5. Draw the plan of a right prism 2 inches long, the horizontal section of which is an equilateral triangle of 2″ side when one of its faces is inclined at 40°, and an edge in that face at 25°. (Case 3.)

6. The plan of the axis of a hexagonal right pyramid (side base 1 inch, height 3 inches), makes an angle of 38° with an *xy* line. Draw the projections of the solid when one edge of the base lies in the horizontal plane, and the plane of the base makes (externally) an angle of 72° with the horizontal plane. N.B.—The solid to be placed with the vertex more distant than the base from the *xy* line. (Case 2.)

7. An octahedron of 2½ inches edge rests on one of its edges in the horizontal plane, and one of the faces containing this edge is inclined at 20°. Draw its plan and an elevation on a vertical plane, making an angle of 40° with the horizontal edge. (Case 2.)

8. A hexagonal right prism of 1½ inch side and 3 inches long rests on one edge in the horizontal plane, and has one of the faces containing that edge inclined at 45°. Draw a plan of

the prism and an elevation on an *xy* line, making an angle of 10° with the plan of the axis of the prism. (Case 2.)

9. A regular hexagonal pyramid (edge of base 1·3 inches, slant edge 3·5 inches) has a face in the horizontal plane. Draw a plan of the solid and an elevation on a ground line, making an angle of 60° with the plan of the axis of the pyramid. (Case 1.)

10. In the pyramid given in the last question draw the plan of the section made by the horizontal plane passing through the centre of the base. Is this plan the true form of the section? (Sections of Solids.)

11. Draw the plan of a right square pyramid (sides of base 2½ inches and height 3½ inches) when two adjacent sides of the base are inclined respectively at 15° and 25°. (Case 4.)

12. In the pyramid given in the last question determine the true form of the section of the pyramid made by a horizontal plane passing through the solid 2½ inches vertically below the apex. (Sections of Solids.)

13. A right prism 3½ inches long, and whose base is a regular hexagon 2½ inches side, rests with one edge of the base horizontal, the face containing that edge being inclined at 55°. Draw its plan and an elevation on a vertical plane, making an angle of 40° with the plan of the axis of the prism. (Case 2.)

14. Contour the octahedron given in Question 7 by horizontal planes at vertical intervals of half an inch, commencing from the level of the plane on which the solid rests. (Sections of Solids.)

15. Contour similarly the solid given in Question 4. (Sections of Solids.)

16. Give sectional elevations of the solid given in Question 11 on three parallel vertical planes, the first passing through the centre of the base, and making an angle of 25° with the plan of one of the sides of the base and the others, one on each side of this plane, and distant horizontally from it half an inch. (Sections of Solids.)

17. Two adjacent faces of an octahedron are inclined at 25° and 70°. The edge of the solid is 2½ inches in length. Draw its plan. (Case 5.)

18. A right pyramid with a regular pentagonal base of 1½ inches, and whose faces make angles of 70° with the base, rests

with its base on a plane inclined at 25°, and one of its faces inclined at 50°. Draw its plan. (Case 5.)

19. Contour this solid at every half inch level, commencing from the lowest point of the solid as it is fixed by the last question. (Sections of Solids.)

20. Two right cylinders interpenetrate, the axes of both being horizontal and in plan including an angle of 75°. The diameters are respectively 2 and 3 inches, their lengths indefinite. Draw the plan of the solids when both rest on the horizontal plane. (Interpenetration of Solids.)

21. A right cone 4 inches high, and whose generatrices are inclined to the axis at 25°, is traversed by a right cylinder 5 inches long, whose axis is horizontal, and meets the axis of the cone at its middle point. The diameter of the cylinder is 1½ inches. Draw a plan of the combined solid and an elevation on a vertical plane, making an angle of 30° with the plan of the axis of the cylinder. (Interpenetration of Solids.)

Defilade Problems and Isometric Projection.

Examples in problems relating to defilade cannot be given without reference to contoured plans of ground, and it would be beyond the scope of the present work to supply such plans.

In Isometric projection also, either the original objects, models of these, or, at least, plans and elevations of the works are required. The student is recommended to take any details of existing designs or works, and give isometric representations of them to some convenient scale. The effect of such drawings is much heightened by shading and colouring on the assumption of some convention regarding the incidence of rays of light. The three equally inclined planes of the cube, for example, might be shaded in three different tints, or, if local colour is to complete the colouring, one plane might be left unshaded, and the other two shaded with two strongly contrasting shade tints. In the latter case, the local colour (representing the materials of which the object is composed) would be washed over the shaded representation for the reasons given in the Hints and Suggestions to Draughtsmen.

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